DEFENCE RESEARCH ESTABLISHMENT
CENTRE DE RECHERCHES POUR LA DÉFENSE
VALCARTIER

DREV - TR-1999-215
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# A MULTIPLE CRITERIA AGGREGATION PROCEDURE FOR THE EVALUATION OF COURSES OF ACTION IN THE CONTEXT OF THE CANADIAN AIRSPACE PROTECTION

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February/février 2001

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## ABSTRACT

Defence Research Establishment Valcartier (DREV) initiated a research activity aimed at investigating and developing advanced technologies, approaches and concepts to provide the 1CAD/CANR Air Operations Centre (AOC) Commander and his senior staff with advisory tools for planning, management and employment of air defence resources and capabilities. An advisor tool was designed to assist the AOC staff managing events and their related Courses of Action (CoAs), as well as prioritising these CoAs according to different evaluation criteria. This command and control tool, called *Commander's Advisory System for Airspace Protection* (CASAP), was foremost developed to deal with events of counter-drug operations: to intercept drug smugglers violating the Canadian airspace, in co-ordination with different services and parties. Since the Commander needs to balance several conflicting and incommensurable criteria to make "*wise*" decisions in such a situation, Multiple Criteria Decision Aid models and procedures were deemed to be appropriate to deal with Canadian airspace protection decision-making situations. This document presents the Multiple Criteria aggregation procedure and the associated analysis tools that were implemented for this advisor tool.

## RÉSUMÉ

Le Centre de recherches pour la défense Valcartier (CRDV) a entrepris une activité de recherche visant à déterminer des approches et des concepts, et à mettre au point des technologies avancées à implanter dans des systèmes conseillers au commandant et au personnel du Centre des opérations aériennes de la Division 1CAD/CANR. Ces systèmes visent à apporter une aide au décideur dans les processus de planification, de gestion et d'utilisation des ressources et des capacités des forces aériennes. Dans le cadre de cette activité, les chercheurs du CRDV ont mis au point le prototype d'un outil conseiller (d'aide à la décision) permettant de soutenir le personnel du Centre des opérations aériennes à gérer des événements et leurs suites d'actions, ainsi que de ranger ces dernières par ordre de priorité. Cet outil de commandement et contrôle, appelé "*Commander's Advisory System for Airspace Protection* (CASAP)", a été mis au point spécifiquement pour traiter des événements lors d'opérations anti-drogues; d'interception de trafiquants de drogues violant l'espace aérien canadien, et ce en coordination avec divers services. Dans ce type de situation, le commandant a besoin d'évaluer les différentes suites d'actions proposées selon plusieurs critères conflictuels et incommensurables afin de prendre la décision qui représente le meilleur compromis. Les modèles et procédures d'aide multicritère à la décision semblent être tout à fait appropriés pour des situations décisionnelles dans le cadre de la protection de l'espace aérien canadien. Ce document présente la procédure multicritère et les outils d'analyse qui ont été implantés dans CASAP.

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## EXECUTIVE SUMMARY

Within 1 CAD/CANR, the Air Operation Centre (AOC) is responsible for the day-to-day air force operations of monitoring and control. For routine operations, command and control decisions are taken through established doctrine, orders, and procedures. As contingency operations are introduced (crisis, contingency deployment or conflict) the AOC becomes the focal point for planning, directing, controlling and monitoring the assigned forces. An important element for either type of operation is the elaboration, mitigation and evaluation of different Courses of Action (CoAs) in order to respond to emergency situations. During such situations, AOC staff members work under stress conditions and have to process a large amount of information within a short time cycle. In fact, time constrains the process to extensively assess the situation, generate wide-range of CoAs, and intensively evaluate them according to significant points of view, before selecting and executing the "best" CoA representing the best possible compromise according to the criteria.

Defence Research Establishment Valcartier (DREV) initiated a research activity aimed at investigating and developing advanced technologies, approaches and concepts to provide the 1CAD/CANR AOC Commander and his senior staff, with advisory tools for planning, management and employment of air defence resources and capabilities. DREV Defence Scientists designed an advisor tool called *Commander's Advisory System for Airspace Protection* (CASAP), to help the AOC staff managing counter-drug events and their related CoAs. This command and control (C&C) tool also helps the Commander (or his representative: the person in charge of the AOC) to screen and prioritise the proposed CoAs to overcome the emergency situation. During, different knowledge acquisition sessions, it was established that in such situations, the Commander needs to balance several conflicting and incommensurable criteria to make "*wise*" decisions. Therefore, Multiple Criteria Decision Aid models and procedures were deemed to be appropriate to deal with Canadian airspace protection decision-making situations. PAMSSEM, a Multiple Criteria aggregation procedure, was then identified and adapted to the C&C requirements in the context of counter-drug operations. PAMSSEM is implemented in CASAP, with different add-ins to help the decision-maker analysing the CoAs, and minimising the risk component introduced by the subjectivity and the uncertainty of the evaluation process. This document presents the Multiple Criteria aggregation procedure and the associated analysis tools that were implemented for this advisor tool.

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## LIST OF ACRONYMS

| | |
|---|---|
| 1 CAD | 1 Canadian Air Division |
| 1 CAD/CANR | 1 Canadian Air Division/Canadian NORAD Region |
| AOC | Air Operation Centre |
| C&C | Command and Control |
| CANR | Canadian NORAD Region |
| CAP | Canadian Airspace Protection |
| CASAP | Commander's Advisory System for Airspace Protection |
| CoA | Course of Action |
| DM | Decision Maker |
| DMS | Decision-Making Situation |
| DREV | Defence Research Establishment Valcartier |
| GAIA | Geometrical Analysis for Interactive Aid |
| MCAP | Multiple Criteria Aggregation Procedure |
| MCDA | Multiple Criteria Decision Aid |
| NORAD | North American Airspace Defence |
| PAMSSEM | Procédure d'Agrégation Multicritère de type Surclassement de Synthèse pour Évaluations Mixtes |

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## LIST OF SYMBOLS

| | |
|---|---|
| P.$\alpha$ | Decision problematic representing a choice of a best alternative |
| P.$\beta$ | Decision problematic representing a sorting of the alternatives into different categories |
| P.$\gamma$ | Decision problematic representing a ranking of the alternatives |
| p.r.s. | Preference relational system |
| **E** | Multiple Criteria Performance Matrix |
| **A** | Set of the courses of actions (CoAs) |
| **F** | Set (family) of criteria or attributes |
| $C_j$ | Criterion *j* |
| $a_i$ | CoA *i* (or alternative *i*) |
| $e_{ij}$ | Evaluation of the $i^{th}$ CoA according to the $j^{th}$ criterion |
| $\sim$ | Indifference relation |
| $\succ$ | Strict preference relation |
| $\succ^{f}$ | Weak preference relation |
| **S** | Outranking relation |
| **?** | Incomparability relation |
| $q_j$ | Indifference threshold for the criterion *j* |
| $p_j$ | Preference threshold for the criterion *j* |
| $v_j$ | Veto threshold for the criterion *j* |
| $\pi_j$ | Relative importance of the $j^{th}$ criterion |
| $\pi'_j$ | Stability interval for the relative importance of the $j^{th}$ criterion |

# 1.0 INTRODUCTION

1 Canadian Air Division/Canadian NORAD Region Headquarters (1 CAD/CANR) Winnipeg is home of the dual headquarters for 1 Canadian Air Division (1 CAD) and the Canadian NORAD Region (CANR). It serves as the central day-to-day Command and Control (C&C) facility for Canada's Air Force overseeing operational air. As CANR headquarters, it monitors the Canadian airspace in support of our commitments to the North American Airspace Defence Command (NORAD). Moreover, 1CAD/CANR develops, directs and co-ordinates Canada's worldwide air force operations using its fleet of fighter, transport, search and rescue, maritime air and tactical helicopter aircraft.

Within 1 CAD/CANR, the Air Operation Centre (AOC) is responsible for the day-to-day air force operations of monitoring and control. For routine operations, C&C decisions are taken through established doctrine, orders, and procedures. As contingency operations are introduced (crisis, contingency deployment or conflict) the AOC becomes the focal point for planning, directing, controlling and monitoring the assigned forces. An important element for either type of operation is the elaboration, mitigation and evaluation of different Courses of Action (CoAs) in order to respond to emergency situations. During such situations, AOC staff members work under stress conditions and have to process a large amount of information within a short time cycle. In fact, time constrains the process to extensively assess the situation, generate wide-range of CoAs, and intensively evaluate them according to significant points of view, before selecting and executing the *"best"*[1] one. Moreover, it might be impossible to have access to the experience of resources that are not inside the command post at that specific time.

Defence Research Establishment Valcartier (DREV) initiated a research activity aimed at investigating and developing advanced technologies, approaches and concepts to provide the 1CAD/CANR AOC Commander and his senior staff, with advisory tools for planning, management and employment of air defence resources and capabilities. DREV defence scientists designed an advisor tool to assist the AOC staff in the management of events and their related CoAs, as well as in the prioritisation of these CoAs according to evaluation criteria. This tool, called *Commander's Advisory System for Airspace Protection* (CASAP), was foremost developed to deal with events of counter-drug operations: to intercept drug smugglers violating the Canadian airspace. Since the Commander needs to balance several conflicting and incommensurable criteria to make "*wise*" decisions in such a situation, Multiple Criteria Decision Aid (MCDA) models and procedures were deemed to be appropriate to deal with Canadian airspace protection (CAP) decision-making situations (DMS).

The readings of Canadian military operation documents (ex. [Ref. 1-5]), and operational checklists, followed by knowledge acquisition sessions with the operational air force personnel led to the identification of five aspects to be considered while evaluating CoAs for counter-drug scenarios in a peacetime context. These aspects were modelled as factors: *Flexibility*, *Complexity*, *Sustainability*, *Cost-of-Resources*, and *Risk*. For each factor, evaluation criteria have been formulated further to a knowledge engineering process [Ref. 6].

The evaluation of the different CoAs according to these criteria provides the basic information (data) to make a decision. A Multiple Criteria Aggregation Procedure (MCAP) is then required to aggregate these data in order to prioritise and identify the best CoA to overcome the threat. Following a comparison of various discrete MCAPs [Ref. 7], it was determined that PAMSSEM ("*Procédure d'Agrégation Multicritère de type Surclassement de Synthèse pour Évaluations Mixtes*")[2] was adequate for a CoA advisor tool [Ref. 8-9].

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[1] "Best" in the sense of the one achieving the best possible compromise.

[2] Multiple Criteria Aggregation Procedure Based on the Outranking Synthesising Approach

The goal of this report is to describe the work done related to the selection and the implementation of a Multiple Criteria Aggregation Procedure in CASAP's prototype for the evaluation and the prioritisation of CoAs in the context of Canadian airspace protection. The presentation of the CASAP prototype and its functionalities is beyond the purpose of this report, which will be reported in a future document. A brief review of the MCAPs reported in the literature and is presented in Chapter 2. The selected MCAP, PAMSSEM, is introduced in Chapter 3, and the settings of its parameters in the context of CAP are described in Chapter 4. Decision-making in a risky context requires performing deep analysis given that the data are imperfect, and that the evaluation process embeds subjectivity. A variety of analysis tools developed for PAMSSEM in the context of CAP is proposed in Chapter 5. Finally, PAMSSEM and its further improvements to deal with a DMS in a context of CAP are discussed in Chapter 6.

This work was carried out at DREV in conjunction with Laval University between January 1997 and September 1999 within Work Unit 3ae12: *Commander's COA Advisory Systems Demonstrators* of project 3ae: *Commander's CoA Advisory Systems.*

## 2.0 REVIEW OF THE MCAPs SUGGESTED IN THE LITERATURE

The Commander's Advisory System for Airspace Protection (CASAP) is a decision-aid prototype based on a Multiple Criteria Aggregation Decision Aid (MCDA) methodology to deal with counter-drug operations. In this context, a set of courses of actions (CoAs) has to be evaluated according to a set of criteria (see Table I). Let **A** be the set of the CoAs, $\mathbf{A} = \{a_1, a_2, \ldots, a_i, \ldots, a_m\}$, and **F**, a coherent family of criteria or attributes[1], $\mathbf{F}=\{C_1, C_2, \ldots, C_j, \ldots, C_n\}$. In the context of Canadian airspace protection (CAP), we defined and formulated 14 different criteria related to five factors ($n$=14, see Table I). These criteria are ranging from cardinal deterministic, to ordinal, distributional, fuzzy, and probabilistic. The value of the $i^{th}$ CoA evaluation according to the $j^{th}$ criterion is denoted $e_{ij}$. The evaluation of all the CoAs according to the set of criteria can be summarised in the Multiple Criteria Performance Matrix **E** (see Table II).

In the context of CAP, the data contained in **E** are mixed, and obtained according to different measurement scales[2] (quantitative or qualitative). A quantitative, also called cardinal, measurement scale is concerned with numbers and quantities (e.g. cost, probabilities, preferences on utility scales, and temperature). A qualitative, also called ordinal, scale measures only the order (e.g. complexity quality and classification grades). Moreover, the evaluation of a CoA according to a criterion could encompass imprecision, subjectivity, and even could be missing. In our modelling of the decision-making situation in the context of CAP, we allow the data contained in the performance matrix **E** to be of diverse types, it could be imperfect, and even missing. Indeed, this way of modelling increases the complexity of the decision-analysis process and the development of CASAP. Nevertheless, we think that it is a accurate way to deal with natural information representing human judgements, values and preferences.

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[1] In this report, we use without distinction attribute and criterion where it is appropriate.

[2] The level of the measurement scale is settled once the criterion is defined.

## TABLE I

Summary of the 14 formulated criteria to evaluation CoAs in context of CAP [Ref.6]

| Factor | Criterion | Optimisation | Scale | Evaluation |
|---|---|---|---|---|
| **Flexibility** | | | | |
| C1: | Covering Operational Tasks | Maximise | Cardinal on [0,1] | Crisp, Deterministic, Continuous |
| C2: | Covering Mission's locations | Maximise | Cardinal on [0,1] | Crisp, Probabilistic, Continuous |
| C3: | Covering Enemy's CoA | Maximise | Cardinal on [0,1] | Crisp, Probabilistic, Continuous |
| **Complexity** | | | | |
| C4: | Operations Complexity | Minimise | Ordinal, 5 echelons | Crisp, Deterministic, Discrete |
| C5: | Logistics Complexity | Minimise | Ordinal, 5 echelons | Crisp, Deterministic, Discrete |
| C6: | C&C Complexity | Minimise | Ordinal, 5 echelons | Distribution, Discrete |
| **Sustainability** | | | | |
| C7: | Sustainability | Maximise | Cardinal, $\mathbf{R}^+$ | Crisp, Deterministic, Continuous |
| **Cost of resources** | | | | |
| C8: | Cost of Resources | Minimise | Cardinal, $\mathbf{R}^+$ | Crisp, Deterministic, Continuous |
| **Risk** | | | | |
| C9: | Impact of Sensors Coverage Gap | Minimise | Ordinal, 3 echelons | Distribution, Discrete |
| C10: | Military personnel loss | Minimise | Ordinal, 7 echelons | Crisp, Probabilistic, Discrete |
| C11: | Collateral damage | Minimise | Ordinal, 7 echelons | Crisp, Deterministic, Discrete |
| C12: | Confrontation risk | Minimise | Ordinal, 7 echelons | Crisp, Probabilistic, Discrete |
| C13: | Equipment reliability | Maximise | Cardinal on [0,1] | Crisp, Probabilistic, Discrete |
| C14: | Personnel effectiveness | Maximise | Ordinal, 5 echelons | Fuzzy, Distribution, Continuous |

## TABLE II

Multiple Criteria Performance Matrix E

$$\begin{array}{cc|ccccc} & & \multicolumn{5}{c}{\textit{Criteria } (1 \ldots n)} \\ & & C_1 & \cdots & C_j & \cdots & C_n \\ \hline & a_1 & e_{11} & \cdots & e_{1j} & \cdots & e_{1n} \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \textit{CoAs } (1 \ldots m) & a_i & e_{i1} & \cdots & e_{ij} & \cdots & e_{in} \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & a_m & e_{m1} & \cdots & e_{mj} & \cdots & e_{mn} \end{array}$$

Subsequent to the evaluation step, the decision-aid process involves an extensive effort of the DM's preference articulation and modelling, which are discussed in the following sub-section.

## 2.1 Decision Maker's Preferences Modelling

Modelling the DM's preferences is not an easy task because it is difficult to apprehend the subjectivity and the imprecision/ambiguity of human behaviour. The major challenge facing the implementation of any decision aid is the "*accurate*" assessment of these preferences. Siskos [Ref. 10] declared that "*... the major problem for the analyst facing multicriterion phenomena raise, is the assessment of a model reflecting at best the preferences of the decision-maker.*"

The literature reviewed reveals many ways and theories about preferences articulation and modelling. For example, utility functions, valued functions, pairwise comparisons, tradeoffs, and discrimination thresholds could be used (see [Ref. 11-17] for more details). In this section, we limit our review to the discrimination thresholds for preferences modelling.

Roy [Ref. 15] maintains that when comparing two CoAs taking into account many criteria, a DM may be in one of the following situations:

i) he/she is indifferent between $a_i$ and $a_k$ (denoted $a_i \sim a_k$),

ii) he/she strictly prefers $a_i$ to $a_k$ (denoted $a_i \succ a_k$),

iii) he/she weakly prefers $a_i$ to $a_k$ (hesitation between indifference and strict preference: denoted $a_i \succ^{f} a_k$), or

iv) he/she considers that $a_i$ is incomparable to $a_k$ (hesitation between $a_i \succ a_k$ and $a_k \succ a_i$, or the two CoAs are *a priori* matchless: denoted $a_i ? a_k$).

It is evident that when considering a single criterion at a time, there are no rooms for incomparability. In classical decision analysis, any criterion is presumed to have an absolute discrimination power. This implies that when comparing two alternatives $a_i$ and $a_k$ according to an absolute criterion (called also "*true-criterion*"), we get the following preference relations (Figure 1):

$$\begin{cases} a_i \succ_j a_k \Leftrightarrow e_{ij} > e_{kj} \\ a_i \sim_j a_k \Leftrightarrow e_{ij} = e_{kj} \end{cases} \tag{1.}$$

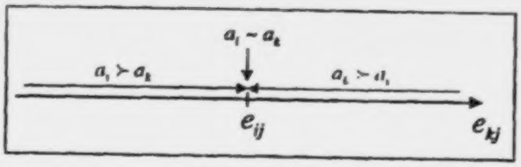

FIGURE 1 - True-criterion

In this case, the indifference is reduced to a single point where the two evaluations are equal, and otherwise we have strict preference situations. The single point indifference is called a strict indifference. However, the evaluation of a CoA with regard to a criterion is often uncertain and imprecise. Moreover, on top of these uncertainties and imprecision, the DM's preferences may involve some hesitations, doubts, indecision, etc. In order to take these behaviours into account, one can introduce an indifference threshold $q_j \geq 0$, such that if the performances of two alternatives on a criterion $j$ (called *quasi-criterion*) differ by less than $q_j$, then there is an indifference relation $\sim_j$ such as (Figure 2):

$$\begin{cases} a_i \succ_j a_k \Leftrightarrow e_{ij} - e_{kj} \geq q_j \\ a_i \sim_j a_k \Leftrightarrow \left| e_{ij} - e_{kj} \right| < q_j \end{cases} \tag{2.}$$

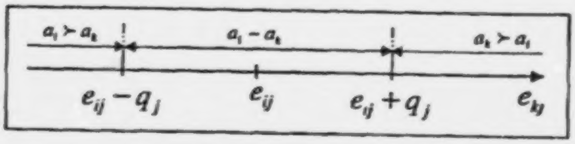

FIGURE 2 - Quasi-criterion

In this case, the hesitation range is expanded from a single point to an interval (which can be variable). If $q_j$ is set to 0, then the *quasi-criterion* collapses to a *real criterion*.

Moreover, one may define a strict preference threshold for a quasi-criterion $j$, $p_j \geq 0$, such that if the performances of $a_i$ and $a_k$ according to this criterion differ by at least

$p_j$, then it is a situation where one alternative is strongly preferred $\succ_j$ to the other one. However, if this difference is between $q_j$ and $p_j$, we can conclude to a weak preference $\succ^f_j$ between the two CoAs. A criterion with preference and indifference thresholds is a *pseudo-criterion*, which is illustrated as follows (Figure 3):

$$\begin{cases} a_i \succ_j a_k \Leftrightarrow e_{ij} \geq e_{kj} + p_j \\ a_i \succ^f_j a_k \Leftrightarrow q_j < e_{ij} - e_{kj} < p_j \\ a_i \sim_j a_k \Leftrightarrow \left| e_{ij} - e_{kj} \right| \leq q_j \end{cases} \qquad (3.)$$
$$\text{where} \quad p_j > q_j$$

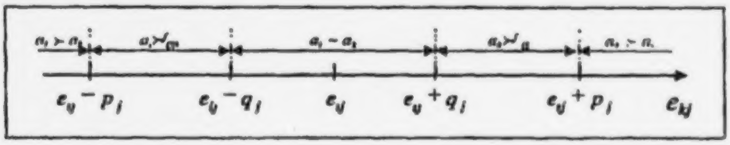

FIGURE 3 - Pseudo-criterion

If $q_j$ is set to 0, then the *pseudo-criterion* collapses to a *pre-criterion*, and if $p_j=q_j$, then the *pseudo-criterion* collapses to a quasi-criterion. The use of constant thresholds for establishing preference relations is illustrated on Figure 3. However, one should note that these thresholds are not necessarily constant; i.e. $q_j = q_j(e_{ij})$ and $p_j = p_j(e_{ij})$.

## 2.2 Multiple Criteria Aggregation Approaches

The DM's preferences and the data contained in **E** should be aggregated in order to recommend the *"best decision"*. During the last three decades, the MCDA discipline has undergone significant development, which gives rise to several aggregation procedures (MCAPs). Each procedure is based on a particular hypothesis and operational approach. The review of the literature reveals that the different known discrete MCAPs are based on one or another of the following operational approaches [Ref. 18]:

- Single criterion synthesising approach: The MCAPs of this approach seek to build an aggregation function $V$ to represent the global score of a CoA $a_i$ by a single synthesising criterion $C(a_i)$ obtained from the scores with regard to criteria $c_1,\ldots,c_n$ by $C(a_i)=V[C_1(a_i),\ldots,C_n(a_i)]$ [Ref. 11-13, 19-22]. This approach aims at constructing a value system that aggregates the DM's preferences on the criteria (attributes) based on strict assumptions; only strict preference and indifference are considered, and these preference relations are complete and transitive. For example, the MultiAttribute (Value) Utility Theory (MAUT/MAVT) drops within the framework of this approach. The DM's preferences are modelled throughout partial utility functions, then aggregated in a single synthesising utility function. This approach is originally known as the American School Approach.

- Outranking synthesising approach: The idea behind this approach is to be able to establish an outranking relation between two CoAs; $a_iSa_k$ : $a_i$ outranks $a_k$ = $a_i$ "is at least as good as" $a_k$ [Refs. 15, 23-28]. This approach is inspired from the social choice theory. Each criterion is considered as a voter, with a particular voting power fixed accordingly with the DM, and each CoA as a candidate. The DM's preferences are modelled throughout a set of parameters (thresholds), and the aggregation is partially compensatory. This aggregation approach is based on the pairwise comparison of the CoAs along each criterion. The preference relational system (p.r.s.) obtained could include the indifference, strict preference, weak preference or even the incomparability. The outranking synthesising approach is associated with the European School, and provides further exploitations and processes to support the DM to conclude to a "*good*" decision.
- Desaggregation-aggregation approach (interactive approach): This approach is not very well defined. It is based on an iterative process with the DM to explore (even generate) different alternatives until a satisfactory one is achieved. Special iterative interactive procedures are used, where the components of the problem and the DM's global judgement policy are

analysed and aggregated into a value system [Refs. 29, 30]. Within this approach, the goal pursued is to help the DMs to improve their knowledge on the problem's state and better articulate their preferences.

- Mixed procedures: These are procedures which combine the outranking and the single criterion synthesising approaches or cannot fit within one of the above categories. These kinds of procedures may lead to different global preference structures.

In classical decision analysis, it is assumed that the outcome is always a choice. In contrast, multiple criteria decision analysis considers explicitly different decisional problematics. According to Roy [Ref. 24], a MCAP outcomes could be one of the following types (the decisional problematic):

- a choice of the best alternative (P.$\alpha$);
- a sorting of the alternatives into different categories (P.$\beta$);
- a ranking of the alternatives (P.$\gamma$) according to their performances;
- a screening of the alternatives, etc.;

## 2.3 Synthesis of some Multiple Criteria Aggregation Procedures

The review of the MCDA literature shows that the aggregation and the exploitation phases of a MCAP can be achieved in different manners. If the MCAP is based on the single criterion synthesising approach, the exploitation step is trivial in the sense that a unique value (according to the unique aggregating criterion) is associated to each CoA during the aggregation phase. Then, no matter what the decisional problemic is, this unique value allows to directly address the proper problemic. However, if the MCAP belongs to the outranking approach, the exploitation phase is performed in the light of the decisional problematic pursued.

The aggregation phase of the MCAP based on the single criterion synthesising approach is often considered as a weighted sum (additive forms such as in these MCAPs:

MAUT, UTA, MACBETH, AHP,...)[1] of the CoA performances obtained according to each criterion. These performances are evaluated by local utility functions, values, priorities (AHP),..., and pondered by scale, harmonisation, or compromise coefficients. Sometimes, the aggregation may be performed using a multiplicative model (such as in these MCAPs: MAUT, AHP,...) or even a mixed model. Generally speaking, the aggregation of this type is compensatory.

The aggregation phase of the MCAPs based on the outranking approach is achieved through the combination of the pairwise comparisons. The pairwise comparison process is performed for each criterion and produces a value for each pair of CoAs of A. The combination of these values takes into account the intrinsic relative importance of the criteria. For these procedures, the aggregation phase consists of a concordance analysis which, most of the time, takes the form of a weighted sum of the pairwise comparison results (such as for these MCAPs: ELECTRE, PROMETHEE, ORESTE, QUALIFLEX,...)[2]. The DM's preferences modelling usually introduces discrimination thresholds: indifference threshold, strict preference threshold, and sometimes a veto threshold. The introduction of these thresholds causes a partial compensation at the level of the concordance analysis. Furthermore, the ELECTRE [Ref. 15, 25] methods introduce a discordance analysis to consider the discordant criteria (those opposed). Moreover, these methods introduced veto thresholds and outranking indices (credibility). These indices are the result of the fusion of the global concordance index to the local discordance indices. This discordance analysis also limits the compensation phenomena. The discrimination and veto thresholds are settled up on measurement scales (natural scales) associated to the criteria. In case of fuzzy evaluations, there are different ways to perform the aggregation phase; for example, it may be based on the operators like the *max*, *min*, *minmax*,...

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[1] See Appendix A for more details.

[2] *Idem*

The great variety of MCAPs makes it quite challenging to choose an appropriate one for a given Decision-Making Situation (DMS). We therefore conducted a comparative study to characterise the properties of various discrete MCAPs in order to determine the most appropriate one for DMS in a context of the CAP. The MCAPs that have been considered for this selection are briefly described in Appendix A. Table III presents the MCAP synthesis achieved according to characteristics developed during a previous study [Ref. 8]. The characteristics used for the selection effort are:

1. *Articulation and modelling of the DM's preferences:* how the DM's preferences are modelled and used in this MCAP :
    - the preference articulation mode: specify if the MCAP uses direct assessment, tradeoffs or pairwise comparisons to help the DM's articulating his preferences;
    - the moment of preference articulation: when the DM is asked to articulate a preference- *a priori*, progressively or *a posteriori* form the use of the MCAP;
    - the global preference relational system (p.r.s.) obtained: $\{\succ, \sim\}$, $\{\succ, \sim, \succ^f\}$, $\{\succ, \sim, ?\}$, $\{S\}$, $\{S, ?\}$;
    - the local preference modelling tools: thresholds (indifference, strict preference, veto), utility/value functions, etc.;
2. *Decision problem type:* the type of decision that may be handled (decision problemics) as defined by Roy [Ref. 24]: a choice of a best alternative (P.$\alpha$), a sorting of the alternatives into different categories (P.$\beta$), a ranking of the alternatives (P.$\gamma$) according to their performances, a screening of the alternatives, etc.;
3. Type of information that each procedure can process:
    - *Nature of eligible data:* levels of the data measurement scales (ordinal (Ord.), cardinal (Card.): ratio or interval);
    - *Characteristics of eligible data:* data can be crisp or represented by distributions (discrete or continuous);

4. *Discriminating power of a criterion:* is each criterion treated with an absolute discrimination power or not;
5. *Level of Compensation :* The compensation is the properties of a MCAP to allow a good performance on one criterion (or many criteria) to counterbalance a poor evaluation(s) on another (or many others);
6. *Inter-criteria information:* Does the MCAP use information to represent the relative importance of each criterion?
7. *Hypothesis:* underlying hypotheses of the MCAP;
8. *MCAP Treatment:* processing algorithm (or calculation algorithm);
9. *Software Package:* whether there is at the present time a computerised support for the method.

## TABLE III

Synthesis of characteristics of some reviewed MCAPs

| MCAP | Descriptors | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Preferences | | | | Decision problem | Information modeling | | | | | Discriminating Power of Criteria | Level of Compensation | Intercriteria Information | Hypothesis[2] | MCAP treatment | Software package |
| | Pref. Articul. Mode | Moment | Pref. Structure (level 2) | Overall r.s.p. | | Measurement Scale Nature | | | Data Characteristics | | | | | | | |
| | | | | | | Ord. | Card. | Mix. | Punc. | Dist. | | | | | | |
| **Basic Procedures** | | | | | | | | | | | | | | | | |
| Weighted Sum | *Direct Assessment* | *a priori* | {≻, ~} | Complete preorder | α | | ✓ | | ✓ | | Absolute | Totally | Total and explicit (Importance Coeff.) | ind., com., inv., tran., dom. | Algebraic Sum | |
| Lexicographical Procedure | *Direct Assessment* | *a priori* | {≻, ~} | Complete preorder | α | ✓ | ✓ | ✓ | ✓ | | Absolute | No | N/A[1] | ind., inv., tran., dom. | Cut diagrams | |
| Conjunctive Procedure | *Direct Assessment* | *a priori* | {≻, ~} | Filtration | *N/A* | ✓ | ✓ | ✓ | ✓ | | Absolute | No | N/A | ind., inv., tran., dom. | Thresholds | |
| Disjunctive Procedure | *Direct Assessment* | *a priori* | {≻, ~} | Filtration | N/A | ✓ | ✓ | ✓ | ✓ | | Absolute | No | N/A | ind., inv., tran., dom. | Thresholds | |
| Maximin Procedure | *Direct Assessment* | *a priori* | {≻, ~} | Complete preorder | α | ✓ | ✓ | | ✓ | | Absolute | No | N/A | ind., inv., tran., dom. | Max & Min Operators | |
| **Procedures for the Single Criterion Synthesising Approach** | | | | | | | | | | | | | | | | |
| TOPSIS | *Direct Assessment* | *a priori* | {≻, ~} | Complete preorder | α | | ✓ | | ✓ | | Absolute | Total | Total and explicit | ind., com., inv., tran., dom. | Euclidean distances | |
| MAVT | Tradeoffs | *a priori* | {≻, ~} | Complete preorder | α | | ✓ | | ✓ | | Absolute | Partial | Total and explicit | ind., inv., tran., dom. | Value aggregation (additive or multiplicative) | |
| UTA | Tradeoffs | *a priori* | {≻, ~} | Complete preorder | α | ✓ | | | ✓ | | Absolute | Partial | Indirect | ind., inv., tran., dom. | Value aggregation (Additive or multiplicative) | ✓ |
| SMART | Tradeoffs and Assessment | *a priori* | {≻, ~} | Complete preorder | α | | ✓ | | ✓ | | Absolute | Partial | Total and explicit | ind., com., inv., tran., dom. | Value aggregation (Additive) | ✓ |
| MAUT | Tradeoffs and Assessment | *a priori* | {≻, ~} | Complete preorder | α | | ✓ | | | ✓ | Absolute | Partial | Total and explicit | ind., inv., tran., dom. | Utility aggregation (Additive or multiplicative) | ✓ |

| MCAP | Descriptors | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Preferences | | | | Decision problem | Information modeling | | | | | Discriminating Power of Criteria | Level of Compensation | Intercriteria Information | Hypothesis² | MCAP treatment | Software package |
| | Pref. Articul. Mode | Moment | Pref. Structure (level 2) | Overall r.s.p. | | Measurement Scale Nature | | | Data Characteristics | | | | | | | |
| | | | | | | Ord. | Card. | Mix. | Punc. | Dist. | | | | | | |
| AHP | Pairwise Comparison | *a priori* | $\{\succ, \sim\}$ and sometimes $\{\succ, \succ^f, \sim\}$ | Complete preorder | $\alpha, \gamma$ | | ✓ | | ✓ | ✓ | Absolute | Partial | Total and explicit | inner and outer ind., inv., dom. | Matrix calculation | ✓ |
| EVAMIX | *Direct Assessment* | *a priori* | $\{\succ, \sim\}$ | Complete preorder | $\alpha, \gamma$ | ✓ | ✓ | ✓ | ✓ | | Absolute | Partial | Total and explicit | Inner and outer Ind., inv., dom. | Weighted sum | ✓ |
| Fuzzy Weighted Sum | *Direct Assessment* | *a priori* | $\{\succ, \succ^f, \sim\}$ | Complete preorder | $\alpha$ | ✓ | ✓ | ✓ | | ✓ | Non-absolute | Total | Total and explicit | ind., com., inv., tran., dom. | α-cut and fuzzy arithm. | |
| Fuzzy Maxmin | *Direct Assessment* | *a priori* | $\{\succ, \succ^f, \sim\}$ | Complete preorder | $\alpha$ | ✓ | ✓ | | ✓ | ✓ | Non-absolute | No | N/A | ind., com., inv., dom. | Max and min operator and fuzzy arithm. | |
| **Procedures for the Outranking Synthesising Approach** | | | | | | | | | | | | | | | | |
| ELECTRE I | Pairwise comparison | *a priori* | $\{S, ?\}$ | Kernel | $\alpha$ | ✓ | ✓ | ✓ | ✓ | | Absolute | Partial | Total and explicit | ind., inv., coal. | Graph theory (kernel) | ✓ |
| ELECTRE II | Pairwise comparison | *a priori* | $\{S^F, S^f, ?\}$ | Partial preorder | $\gamma$ | ✓ | ✓ | ✓ | ✓ | | Absolute | Partial | Total and explicit | ind., inv., coal. | Graph theory (Distillation) | |
| ELECTRE III | Pairwise comparison | *a priori* | $\{S, ?\}$ valued | Partial preorder | $\gamma$ | ✓ | ✓ | ✓ | ✓ | | Non absolute | Partial | Total and explicit | ind., inv., coal. | Graph theory (distillation) | ✓ |
| ELECTRE IV | Pairwise comparison | *a priori* | $\{S^1, S^2, S^3, S^4, S^5, ?\}$ | Partial preorder | $\gamma$ | ✓ | ✓ | ✓ | ✓ | | Non absolute | Partial | N/A² | ind., inv., coal. | Graph theory (distillation) | ✓ |
| ELECTRE IS | Pairwise comparison | *a priori* | $\{S, ?\}$ | Partial preorder | $\alpha$ | ✓ | ✓ | ✓ | ✓ | | Non absolute | Partial | Total and explicit | ind., inv., coal. | Graph theory (core) | ✓ |
| ELECTRE SORT | Pairwise comparison | *a priori* | $\{S, ?\}$ | Partial interval order | $\beta$ | ✓ | ✓ | ✓ | ✓ | | Non absolute | Partial | Total and explicit | ind., inv., coal. | Disjunctive and conjunctive | ✓ |
| PROMETHE I | Pairwise comparison | *a priori* | $\{\succ, \sim, ?\}$ valued | Partial preorder | $\gamma$ | ✓ | ✓ | ✓ | ✓ | | Non absolute | Partial | Total and explicit | ind., inv., coal. | Graph theory (flux) | ✓ |
| PROMETHE II | Pairwise comparison | *a priori* | $\{\succ, \sim\}$ valued | Complete preorder | $\gamma$ | ✓ | ✓ | ✓ | ✓ | | Non absolute | Partial | Total and explicit | ind., inv., coal. | Graph theory (flux) | ✓ |
| MELCHIOR | Pairwise comparison | *a priori* | $\{S, ?\}$ valued | Partial preorder | $\gamma$ | ✓ | | | ✓ | | Non absolute | Partial | Complete order | ind., inv. | Graph theory (distillation) | |
| ORESTE | Pairwise comparison | *a priori* | $\{\succ, \sim, ?\}$ valued | Partial preorder | $\gamma$ | ✓ | | | ✓ | | Absolute | Partial | Complete preorder | ind., inv., coal. | Graph theory | ✓ |
| REGIME | Pairwise comparison | *a priori* | $\{S, ?\}$ | Partial preorder | $\gamma$ | ✓ | | | ✓ | | Absolute | Partial | Complete preorder | ind., inv. | Graph theory | ✓ |
| NAIADE | Pairwise comparison | *a priori* | $\{S, ?\}$ | Partial or complete preorder | $\gamma$ | ✓ | ✓ | ✓ | ✓ | ✓ | Non absolute | Partial | N/A | ind., inv. | Fuzzy arithm. and flux results | ✓ |

| MCAP | Descriptors | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Preferences | | | | Decision problem | Information modeling | | | | | Discriminating Power of Criteria | Level of Compensation | Intercriteria Information | Hypothesis[2] | MCAP treatment | Software package |
| | Pref. Articul. Mode | Moment | Pref. Structure (level 2) | Overall r.a.p. | | Measurement Scale Nature | | | Data Characteristics | | | | | | | |
| | | | | | | Ord. | Card. | Mix. | Punc. | Dist. | | | | | | |
| PAMSSEM I | Pairwise comparison | *a priori* | $\{S, ?\}$ valued | Partial preorder | $\gamma$ | ✓ | ✓ | ✓ | ✓ | ✓ | Non absolute | Partial | Total and explicit | Ind., inv. | Graph theory (flux) | |
| PAMSSEM II | Pairwise comparison | *a priori* | $\{S, ?\}$ valued | Complete preorder | $\gamma$ | ✓ | ✓ | ✓ | ✓ | ✓ | Non absolute | Partial | Total and explicit | Ind., inv. | Graph theory (flux) | |
| **Ordinal Procedures and Combined Procedures** | | | | | | | | | | | | | | | | |
| QUALIFLEX | Pairwise comparison | *a priori* | $\{S, ?\}$ | Semiorder | $\gamma$ | ✓ | | | ✓ | | Absolute | Partial | Total or partial and explicit | ind., inv. | Concordance analysis | ✓ |
| Lexicography of Semiorders | Direct assessments | *a priori* | $\{\succ, \succ', \sim\}$ | N/A[1] | $\alpha$ | | | ✓ | | ✓ | Absolute | No | N/A | Ind., Inv., tran., dom. | Possibility and Necessity Measurements | |
| Borda | Pairwise comparison | *a priori* | $\{S, ?\}$ | Complete preorder | $\gamma$ | ✓ | ✓ | ✓ | | ✓ | Non absolute | Partial | Total and explicit | Ind., inv., coal. | Graph theory | |

[1] N/A: Not Applicable,

[2] ind.: independence, com.: commensurability, inv.: invariance, tran.: transitivity, dom.: dominance; coal.: coalition

## 2.4 Selection of a MCAP for CASAP

In order to select an appropriate MCAP to deal with the DMS in a context of CAP, one should match the MCAP features to the decisional context requirements. In the context of violation of Canadian airspace, the requirements of the CASAP could be summarised as follows:

- Evaluations are mixed: ordinal and cardinal, crisp and distribution;
- Decision problematics is P.$\gamma$: prioritising (ranking) the CoAs;
- DM wants to assign a relative importance coefficient to balance the criteria;
- The DMS requires a "controlled" compensation between the criteria;
- Decision-making in a risky context: model the uncertainties, DM's hesitations and preferences accordingly to the doctrine and the rules of operations;
- Limited time to evaluate and prioritise the CoAs (in case of emergency);
- Possibilities of missing information;
- The DM is well identified (single decision-maker).

The majority of MCAPs proposed in the literature implicitly consider the hypothesis that the nature and the type of data contained in **E** (representing the evaluations of the CoAs) are relatively homogenous. The MCAP to be implemented in CASAP should limit the compensation between the criteria, deal with their heterogeneity, and their measurement scales. Moreover, the thresholds should help the DMs articulate and distinguish their preferences. Thus, a MCAP based on an outranking approach is suitable to deal with the DMS in the context of CAP. This approach allows controlling the compensation phenomena among the criteria. Among the different MCAPs, PAMSSEM seemed to be the most appropriate for CASAP. In fact, there are PAMSSEM I and PAMSSEM II. The two procedures are based on the same aggregation phase, but differ in the exploitation phase. PAMSSEM II leads to a total pre-order[1] of the CoAs. However, PAMSSEM I produces a partial pre-order, which means that in some situation it will be impossible to discriminate some CoAs without introducing new information. In this document, we use PAMSSEM to refer to both versions when appropriate. PAMSSEM II is the procedure actually implemented in CASAP's prototype.

PAMSSEM was designed to deal with mixed and missing evaluations. It offers enough flexibility to consider the DM's preferences and to control the compensations. PAMSSEM amalgamates the ELECTRE III aggregation phase to PROMETHEE I and II exploitation. The ELECTRE III aggregation allows a high level of subtleties to build a valued outranking relation, where PROMETHEE I and II offer a simple way to exploit this relation.

It is important to mention that the implemented PAMSSEM II in CASAP was adapted and improved. Additionally, CASAP includes a series of add-ins specially developed to take advantage of PAMSSEM II. In the following chapters, we present PAMSSEM with its add-ins.

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[1] A pre-order is a total ranking of the CoAs with possibility of equivalencies (reflexive and transitive)

## 3.0 MULTIPLE CRITERIA AGGREGATION PROCEDURE FOR CASAP: PAMSSEM I AND II

A preliminary version of PAMSSEM I and II was developed by Martel *et al.* [Ref. 31] to deal with mixed evaluations: ordinal, cardinal, punctual, distributional (random, fuzzy, linguistic), as well as missing data. PAMSSEM belongs to the outranking approach and encompasses two phases: the aggregation and the exploitation. This MCAP is based on ELECRE III [Ref. 32] for the construction of the outranking relation, on PROMETHEE I and II [Ref. 33] for the exploitation of these relations, and on NAIADE [Ref. 34] for the manipulation of fuzzy evaluations. The purpose of PAMSSEM is to construct an outranking relation, and provide further exploitation to recommend a "*good*" alternative to the DM. The construction of an outranking relation is based on a formal modelling of the DM's preferences according to each criterion: the local preferences modelling phase.

Like ELECTRE III, PAMSSEM uses discrimination thresholds (indifference and strict preference) with different criteria. The indifference threshold represents the highest difference between two evaluations according to a same criterion for which the DM is incapable to make a clear choice, given that everything is the same otherwise. The strict preference threshold represents the smallest difference between two evaluations according to a same criterion for which the DM is able to make a clear preference for one, given that everything is the same otherwise. Thus, PAMSSEM considers criteria with limited discrimination power; not all the criteria are treated as a "*true-criterion*". PAMSSEM's inter-criteria information is of two types: explicit set of coefficients representing the relative importance of the criteria, and veto thresholds ($v_j$) for some criteria where it is appropriate. The veto threshold represents a limit of tolerance that DMs are willing to accept for any compensation. In other words, they may refuse to choose a CoA $a_i$ over $a_k$, if the performance of $a_k$ is higher than the performance of $a_i$ according to the criterion $j$ by at least $\nu_j$.

PAMSSEM I and II allow to deal with different natures and different types of data. In fact, the evaluations can be crisp quantities, discrete/continues random variables (with mass functions or density probability functions $f_{ij}(x_{ij})$), linguistic or fuzzy numbers (which are associated with membership functions $\mu_{ij}(x_{ij})$). PAMSSEM also deals with missing evaluations that could be the result of lack of knowledge, forgetfulness or inappropriateness.

### 3.1 Treatment of Distributional Evaluations

It is possible to have an evaluation $e_{ij}$ which is a distribution representing a random variable (or statistical variable) $X_{ij}$ with a probability distribution function or a density function $f_{ij}(x_{ij})$. Note that the crisp evaluations can be seen as a degeneration of a distribution; the distribution is reduced to only one point ($P(X_{ij} = e_{ij})=1$). The random variables could be continue or discrete. It is easy to deal numerically with discrete functions. However, the continuous variables are chopped up into discrete functions using a discretisation step. Therefore, both continuous and discrete variables will be uniformly processed. Briefly, when the evaluation according to the $j^{th}$ criterion is represented by a continuous distribution, PAMSSEM chops up the mass function according to a discretisation step. It is extremely important to set this step to a value smaller than the indifference threshold $q_j$ (for example $^{q_j}/_{10}$). By doing so, it is possible to take into account the preference hesitations represented by the discrimination thresholds during the processing. A discrete function is a histogram obtained by chopping up the continuous function. The base of each rectangle of the histogram is centred on a crisp value with a probability represented by the surface of this rectangle.

In the context of CAP, the knowledge engineering and modelling processes led to the formulation of many criteria producing distributional evaluations. For example, the evaluations obtained according to the criteria C&C complexity ($C_6$), and impact of sensor coverage gap ($C_9$) are distributions. Here, the distributions represent the imprecision and uncertainties. The density functions may be uniform, triangular, Gauss function, lognormal, etc. However, in the case of the criterion ($C_{14}$) evaluating the personnel's effectiveness, the distribution represents ambiguities and vagueness of a linguistic evaluation. This evaluation is obtained using an ordinal scale with five levels associated to $C_{14}$: very weak reliability, weak reliability, reliable, very reliable, and very highly reliable. In the reported applications in the decision-aid literature, the ordinal evaluations are usually ignored by transforming this information in a cardinal one, simply by matching numbers to the ordinal evaluations. This way of doing is prohibited by the measurement theory [Ref. 35-39].

In CASAP, we tried to develop a decision-aid tool that can properly deal with the different measurement levels. We think that the fuzzy set theory can better model the linguistic evaluations. For example, suppose that the evaluations of two CoAs according to $C_{14}$ are respectively (reliable) and (very reliable). Linguistic evaluations such as these evaluations can be modelled using two membership functions as shown by Figure 4. In case of fuzzy quantities (numbers), one may use L-R[1] membership functions as shown by Figure 5.

With fuzzy evaluations (e.g. linguistic evaluation) represented by a membership function, a processing similar to the one suggested in NAIADE [Ref. 34] can be adopted. This treatment consists in "*rescaling the ordinates of a membership function*". One could defines a function $f_{ij}(x_{ij}) = K_{ij} . \mu_{ij}(x_{ij})$, in such way to obtain a probability density function verifying the following condition:

$$\int_{-\infty}^{\infty} f_{ij}(x_{ij})dx_{ij} = 1 \qquad (4.)$$

---

[1] L-R: left-right

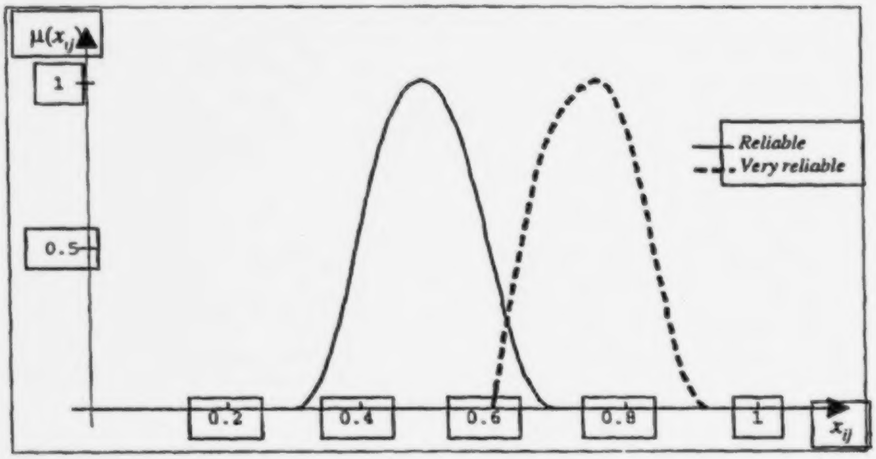


FIGURE 4 - Linguistic evaluations

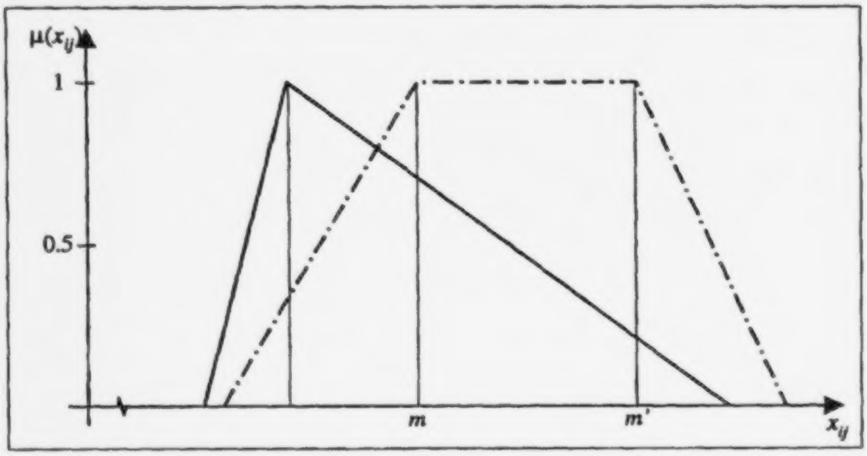


FIGURE 5 - L-R Fuzzy numbers

### 3.2 PAMSSEM Aggregation Phase

The aggregation phase of PAMSSEM begins by computing a **concordance index** $C(a_i,a_k)$ for each pair of CoAs $(a_i,a_k)\in A\times A$. This index is obtained as follows:

$$C(a_i,a_k)=\sum_{j=1}^{n}\pi_j\cdot\delta_j(a_i,a_k)=\sum_{j=1}^{n}\pi_j\cdot\delta_j(e_{ij},e_{kj}) \qquad (5.)$$

where $\pi_j$ is a normalised scalar representing the relative importance of the $j^{th}$ criterion; $\sum_{j=1}^{n} \pi_j = 1$. $\delta_j(a_i, a_k)$ is a local outranking index computed for each pair of CoAs according to each criterion as follows:

$$\delta_j(a_i, a_k) = \sum_{x_{kj}} \left( \sum_{x_{ij}} \bar{\delta}_j(x_{ij}, x_{kj}) \cdot f_{ij}(x_{ij}) \right) \cdot f_{kj}(x_{kj}) \qquad (6.)$$

$f_{ij}(x_{ij})$ and $f_{kj}(x_{kj})$ are respectively the probability density functions (discrete) of $x_{ij}$ and $x_{kj}$. In the case of crisp evaluation, we obtain $P(X_{ij} = x_{ij}) = f_{ij}(x_{ij}) = 1$. $\bar{\delta}_j(x_{ij}, x_{kj})$ is an index computed according to the following formula (like in ELECTRE III):

$$\bar{\delta}_j(x_{ij}, x_{kj}) = \begin{cases} 1 & \text{if } -q_j \le \Delta_j \\ -\dfrac{\Delta_j - p_j}{p_j - q_j} & \text{if } -p_j < \Delta_j < -q_j \\ 0 & \text{if } \Delta_j \le -p_j \end{cases} \qquad (7.)$$

where $\Delta_j = x_{ij} - x_{kj} \approx e_{ij} - e_{kj}$, and $0 \le q_j \le p_j \le E_j$; $E_j$ is the maximum range of the measurement scale of the $j^{th}$ criterion; $q_j = q_j(x_{kj})$ and $p_j = p_j(x_{kj})$. The crisp evaluations can be handled by considering $f_j(x_{ij})=1$ if $x_{ij}=e_{ij}$ and 0 otherwise.

From a purist point of view, it is required to consider an ordinal criterion as a true-criterion. In this case, we have $q_j = p_j = 0$. The local concordance index is then obtained as follows:

$$\delta_j(x_{ij}, x_{kj}) = \begin{cases} 1 & if \quad 0 \le \Delta_j \\ 0 & if \quad 0 > \Delta_j \end{cases} \qquad (8.)$$

For the sake of processing uniformity for the cardinal and ordinal evaluations, we suggest a slight modification in the computation of the concordance index for the ordinal criteria (when the number of levels of the ordinal scale >3) as follows:

$$\delta_j(x_{ij}, x_{kj}) = \begin{cases} 1 & if \quad 0 \le \Delta_j \\ \frac{1}{2} & if \quad -1 \le \Delta_j < 0 \\ 0 & if \quad \Delta_j < -1 \end{cases} \qquad (9.)$$

$\Delta_j$ is the inter-level difference. The measurement theory proscribes the computation of this difference for ordinal scales. However, without the suggested modification, the ordinal criteria will be treated as true-criteria, which will give these criteria more power within the aggregation process. It is then *acceptable* in the context of CAP to consider the information obtained according to these criteria as slightly more than ordinal.

The aggregation phase also involves the computation of another index: a **local discordance index**. This index is computed for each criterion and for each pair of CoAs $(a_i,a_k)\in \mathbf{A}\times\mathbf{A}$. The local discordance index $D_j(a_i,a_k)$ states the opposition of the criterion $j$ to the assertion that $a_i$ outranks $a_k$. This index is computed according to the following formula:

$$D_j(e_{ij},e_{kj}) = \sum_{x_{ij}}\left(\sum_{x_{kj}} \overline{D}_j(x_{ij},x_{kj})\cdot f_{kj}(x_{kj})\right)\cdot f_{ij}(x_{ij}) \qquad (10.)$$

Where $D_j(a_j,a_k) = D_j(e_{ij},e_{kj})$; and

$$\overline{D}_j(x_{ij},x_{kj}) = \begin{cases} 0 & \text{if } -p_j \le \Delta_j \\ -\dfrac{(\Delta_j + p_j)}{\nu_j - p_j} & \text{if } -\nu_j \le \Delta_j < -p_j \\ 1 & \text{if } \Delta_j \le -\nu_j \end{cases} \qquad (11.)$$

$\nu_j=\nu_j(x_{kj})$ is the veto threshold; $\nu_j>p_j$. Note that the value of this threshold is influenced by the importance of the $j^{th}$ criterion. The veto threshold $\nu_j$ is introduced to control the level of compensation. For example, to intercept a drug smuggler, the Commander has to consider two CoAs: $a_i$ and $a_k$. The CoA $a_i$ involves largely more losses of lives than $a_k$. A veto threshold on the criterion *risk of losses of lives* will prevent $a_i$ to globally outrank $a_k$, even if $a_i$ is at least as good as (even better than) $a_k$ on the other criteria. The veto threshold may vary with the position of the evaluation on the measurement scale associated to the criterion $j$ ($\nu j$ $[e_{ij})$]).

From a purist point of view, it is difficult to compute discordance indices or define a veto threshold in the case of ordinal criteria (true criteria). Here again, if we want to assure a uniform processing for the ordinal and cardinal evaluations, we suggest computing local discordance index for the ordinal criterion as follows:

$$\overline{D}_j(x_{ij},x_{kj})=\begin{cases}0 & \text{if } -\frac{\ell_j+1}{2}\leq\Delta_j\\ \min\left[1,-\xi(\pi_j)\cdot(\Delta_j+\frac{\ell_j+1}{2})\right] & \text{if } \Delta_j<-\left(\frac{\ell_j+1}{2}\right)\end{cases} \qquad (12.)$$

where $\ell_j$ is the number of levels of the measurement scale associated with the $j^{th}$ ordinal criterion; $\ell_j>3$. $\Delta_j$ is the inter-level difference, and $\xi(\pi_j)$ is a non-decreasing function of the relative importance of the $j^{th}$ criterion. This function could be, for instance, linear or exponential; $\xi(\pi_j) = 0.2\ (1 + \pi_j/2)$, or $\xi(\pi_j) = 0.2\ (2 + e^{\pi j/2})$,…

Then, PAMSSEM aggregates the concordance and the local discordance indices to establish an **outranking degree** $\sigma(a_i,a_k)$ for each pair of CoAs $(a_i,a_k)\in A\times A$. As suggested by Rousseau and Martel [Ref. 39], these degrees can be obtained as follows:

$$\sigma(a_i,a_k)=C(a_i,a_k)\cdot\prod_{j=1}^{n}\left[1-D_j^3(a_i,a_k)\ \right]\Rightarrow 0\leq\sigma(a_i,a_k)\leq 1 \qquad (13.)$$

$\sigma(a_i, a_k)$ represents a "*consistency*" level of the conclusion that "the CoA $a_i$ globally outranks $a_k$", taking into account all the evaluation criteria. For example, if $\sigma(a_i, a_k)=1$, then the conclusion "$a_i$ outranks $a_k$" is very well established.

The set of valued outranking relations obtained can be represented in an *outranking matrix* (see Table IV). It is important to mention that the aggregation phase of PAMSSEM authorises the possibility to have incomparability; it is possible that at the same time $a_i$ outranks $a_k$, and $a_k$ outranks $a_i$. This result is mainly due to the introduction the discrimination and veto thresholds.

TABLE IV

Outranking matrix

| | $a_1$ | $a_2$ | ... | $a_m$ |
|---|---|---|---|---|
| $a_1$ | - | $\sigma(a_1, a_2)$ | ... | $\sigma(a_1, a_m)$ |
| $a_2$ | $\sigma(a_2, a_1)$ | - | ... | $\sigma(a_2, a_m)$ |
| .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. |
| $a_m$ | $\sigma(a_m, a_1)$ | $\sigma(a_m, a_2)$ | ... | - |

## 3.3 PAMSSEM Exploitation Phase

The exploitation of the outranking relations consists in making a synthesis of this outranking matrix to provide a recommendation. In CASAP, this recommendation will consist in ranking the CoAs. This exploitation may be obtained by introducing the concept of entering and leaving flows of PROMETHEE. For each CoA $a_i$, we compute its leaving flow $\sigma^{+}(a_i)$ and its entering flow $\sigma^{-}(a_i)$ as follows:

$$\sigma^{+}(a_i) = \sum_{\forall a_k \neq a_i} \sigma(a_i, a_k) \tag{14.}$$

$$\sigma^{-}(a_i) = \sum_{\forall a_k \neq a_i} \sigma(a_k, a_i) \tag{15.}$$

The leaving flow represents the overall relative strengths of the CoA $a_i$, and the entering flow represents its overall relative weaknesses. Note that the values of these flows can change if a CoA is introduced or removed to/from the set A. On the basis of these two flows, it is possible to obtain a partial or a total preorder. In PAMSSEM I, a partial pre-order is obtained by the following procedure:

- Rank the CoAs of A according to a decreasing order of $\sigma^{+}(a_i)$: this ranking constitutes a first complete preorder $P^{+}$ of the CoAs;

- Rank the CoAs of A according to an increasing order of $\sigma^-(a_i)$: this ranking constitutes a second complete preorder $P^-$ of the CoAs;
- A partial preorder $P$ is obtained by the intersection of these two complete preorders: $P = P^+ \cap P^-$

$P$ can be a total pre-order if and only if $P^+= P^-$. PAMSSEM I leads to a ranking including incomparability. This means that for some CoAs it is difficult to clearly indicate a preference or an indifference situation. PAMSSEM II overcomes these difficulties by deriving a total preorder. This preorder is the result of computing for each CoA $a_i$ a net flow $\Phi(a_i) = \sigma^+(a_i) - \sigma^-(a_i)$, and then ranking the CoAs of A according to a decreasing order of the net flow. As an illustration, Let A=$\{a_1, a_2, a_3, a_4, a_5\}$ a set of 5 CoAs with the flows of Table V computed by PAMSSEM. PAMSSEM I and II will produce the rankings presented in Figure 6.

TABLE V

Result of PAMSSEM I and II

| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| $\sigma^+(a_i)$ | 0.4 | 1.8 | 1.0 | 0.1 | 0.7 |
| $\sigma^-(a_i)$ | 0.6 | 0.2 | 0.4 | 1.6 | 1.2 |
| $\Phi(a_i)$ | -0.2 | 1.6 | 0.6 | -1.5 | -0.5 |

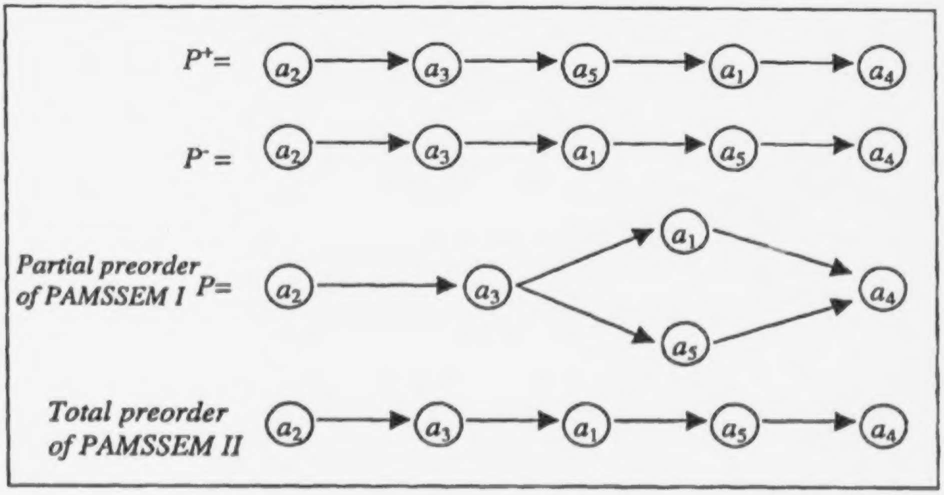

$P^+=$
$a_2$
$a_3$
$a_5$
$a_1$
$a_4$
$P^-=$
$a_2$
$a_3$
$a_1$
$a_5$
$a_4$
Partial preorder of PAMSSEM I
$P=$
$a_2$
$a_3$
$a_1$
$a_5$
$a_4$
Total preorder of PAMSSEM II
$a_2$
$a_3$
$a_1$
$a_5$
$a_4$

FIGURE 6 - Example of PAMSSEM I and II ranking

### 3.4 Treatment of Missing Values

In addition of mixed evaluations, PAMSSEM I and II allow to consider missing evaluations. We might have to deal with missing evaluations of two types: *i*) non-relevant, or *ii*) relevant but impossible to obtain. In the first case, we try to make sure that these "*holes*" in the performance matrix **E** do not favour or disfavour the concerned CoAs. For example, if the evaluation $e_{ij}$ is missing, then we have

$$\Delta_j(a_i,a_k)=\Delta_j(a_k,a_i)=0,\ \forall a_k \in \mathbf{A} \qquad (16.)$$

This implies:

$$\begin{cases} \delta_j(a_i,a_k)=\delta_j(a_k,a_i)=1 \\ D_j(a_i,a_k)=D_j(a_k,a_i)=0 \end{cases},\ \forall a_k \in \mathbf{A} \qquad (17.)$$

In fact, we replace each missing evaluation $e_{ij}$ by the one of the CoA $a_k$ ($e_{kj}$) that is compared with it.

In the second case (relevant missing evaluations), instead of replacing the missing evaluation by the mean-value of the other CoAs evaluations according to this criteria (as it is regularly done in statistics), we replace the missing evaluation by a distribution of evaluations retrieved from the other (none missing) CoAs evaluations. For example, if the evaluation $e_{ij}$ is missing, we replace it by the set $\{e_{1j}, e_{2j}, \ldots, e_{kj}, \ldots, e_{mj}\}$ for all the CoAs $a_k \in \mathbf{A} \setminus \{a_i\}$; i.e. the evaluation of $a_i$ will be modelled by a statistic variable $X_{ij}$ with all the values of $e_{kj}$, $a_k \in \mathbf{A} \setminus \{a_i\}$ and a mass function $f(x_{ij}) =(1/(m-1)$ ($m$ is the number of CoAs in **A**).

## 4.0 SETTING PAMSSEM'S PARAMETERS

In order to prioritise (rank) the different CoAs, PAMSSEM needs the data contained in the multiple criteria performance table **E**, as well as the parameters representing the DM's preferences. These parameters are the discrimination thresholds, the veto threshold, and the relative importance of each criterion. In this chapter, we present different procedures to set the values of these parameters.

Some criteria considered in CASAP have ordinal measurement scales ($C_4$, $C_5$, $C_6$, $C_9$, $C_{10}$, $C_{11}$, and $C_{12}$). From a purist point of view, these criteria should be considered as true criteria. An ordinal criterion forbids the computation of differences between the CoAs evaluations. Then, there is no need to set the values of the thresholds for these criteria. PAMSSEM proposes a special treatment for these criteria, and default settings are already implemented. However, the criteria ($C_1$, $C_2$, $C_3$, $C_7$, $C_8$, and $C_{13}$) are cardinal and treated as pseudo-criteria (with thresholds). Then it is important to set these thresholds. On another front, we also have to set the relative importance of the criteria as well as the veto thresholds. In the following sections, we will present methods to assign default values to the criteria thresholds that can be modified and fine-tuned by the user. Then we suggest a comprehensive method to set the relative importance of the criteria.

### 4.1 Setting Thresholds

PAMSSEM introduces two types of discrimination thresholds for each criterion: indifference threshold $q_j$ and preference threshold $p_j$. It is not an easy task to assign "*right*" values to the thresholds. These thresholds must represent the DM's preferences, which are very difficult to apprehend due to the subjectivity and the imprecision/ambiguity of human behaviour. These thresholds translate on the one hand the sensitivity of the DM to the difference between the evaluations of the pair of CoAs, and on the other hand the imprecision/ambiguity and subjectivity of the evaluation process [Ref. 40]. For these reasons we think that these parameters are introduced to take into account *at the same time* the imperfection of the information and the DM's preferences (hesitations).

Note that these thresholds introduce a very flexible way to model the nuances of the DM's preferences. However, the major challenge facing the implementation of any MCAP is the "*accurate*" assessment of these parameters. This difficulty is not inherent to the MCDA field, but is about any decision-aid methodology [Ref. 11]. When modelling the DM's preferences, one should keep in mind what Li and Adams [Ref. 41] reported: *"... one of the most perplexing aspects of human decision-making behaviour under risk is the sensitivity of preferences to seemingly minor changes in the way a problem is presented"*.

Setting the thresholds is considered as one of the difficulties of the implementation of a MCDA methodology. Users of decision aids have general misunderstandings of the roles and meanings of these thresholds. They face difficulties to set proper values for these parameters; in many situations, they may simply ignore these parameters. Roy and Bouyssou [Ref. 15] maintain that it is better to set imperfect values than ignoring them. One can implement within CASAP default settings for these thresholds. The default values should be variable, depending on the type of the event considered. In order to determine proper default values, one should perform an extensive study of the DMS in order to identify the variability of the events facing the DM. After analysing the ways of response to each one, in a close work with the DM and according to the doctrine, one could determine default values for each type of event. In this section, we suggest default values only for one type of events, the CAP. However, a validation should be carried out in order to make sure that these values represent the DM's preferences.

### 4.1.1 Indifference Thresholds

The values of the thresholds should be determined in a co-operative way with the DM (in this case the Commander or the person in charge of the AOC). The DMs can be asked to assess their indifference threshold for each criterion when confronted by the choice between two CoAs based only on the evaluations according to the selected criterion[1]. This threshold may be modulated according to the values of the evaluations; for example a difference of 10 units if the evaluations are 100 and 110 has not the same impact if these evaluations are respectively 10000 and 10010. It is then possible to define variable indifference threshold in function of the positions of the evaluations on the measurement scale. This first part of the indifference threshold represents the DM's preference sensitivity. Yet, it is necessary to add the effect of the information imperfections to this threshold. Within CASAP, the imperfection component is already taken into account in the evaluation process. In fact, many imperfection-modelling technologies were used in the criteria formulation (ex. fuzzy theory, probabilities, etc.). Then there is no need to increase the value of the indifference threshold to consider the data imperfection component when it is modelled otherwise.

Keep on mind that the indifference threshold $q_j$ represents the highest difference between the evaluations of two CoAs according to a criterion $j$ for which the DM is incapable to make a clear choice between these two CoAs, given that everything is the same otherwise. In order to make a direct estimate of this threshold for each criterion, one should suggest to the DMs a large value for this difference and then decrease it progressively until they may not make a distinction between the choice of one or another of the two CoAs. Then, it is necessary to begin with a small value of this difference and increase it progressively until the value of the difference makes sense for the DM. It is also possible to guess this threshold using a simulation method as proposed by Rogers and Bruen [Ref. 40].

### 4.1.2 Strict Preference Threshold

The value of the preference threshold $p_j$ represents the smallest difference between the evaluations of two CoAs according to the criterion $j$ for which the DM is able to make a clear choice of one, given that everything is the same otherwise. To assess directly this threshold, one may begin by suggesting a value for this difference slightly higher than $q_j$. Then, one increases progressively this value until the DM is able to indicate a clear choice of a CoA (*without any hesitation*). According to Roy et *al.* [Ref. 42] the value of $p_j$ should be substantially higher than $q_j$. Here again, a simulation method might help setting the values of this threshold.

---

[1] Assess the largest accepted difference of evaluations for which the DM may accept to choose without distinction one or another

### 4.1.3 Veto Threshold

To determine the value of the veto thresholds for some criteria, one can suggest to the DMs a value higher than $p_j$, and then increases it progressively until they declare that this value constitutes a limit of tolerance for any compensation. In other words,they may refuse to choose a CoA $a_i$ over $a_k$, if the performance of $a_k$ is higher than the performance of $a_i$ according to the criterion $j$ by at least $v_j$. Exceptionally $v_j$ may have the same value as $p_j$. In this case, we talk about a dictatorial criterion. The value of $v_j$ is usually set to at least three times the value of $p_j$ [Ref. 40]. Note that the usage of the veto threshold should be limited to the most "*critical*" criteria. This threshold strengthens the power of a particular criterion.

## 4.2 Setting Default Values for Thresholds

As mentioned in the previous section, the setting of the thresholds is dependent on three factors related to the criteria: imperfection of the data (evaluations), the DM's preferences (hesitations) as well as the relative importance of the criteria. For each criterion, we define three levels of the data imperfection ($I_j$) as follows:

- Low level of imperfection (uncertainty/imprecision/subjectivity): L;
- Moderate level of imperfection (uncertainty/imprecision/subjectivity): M;
- High level of imperfection (uncertainty/imprecision/subjectivity): H.

---

CoA.

For each criterion *j*, we determine default values for the thresholds $q_j$, $p_j$ and $v_j$ by considering the data imperfection level ($I_j$) and the relative importance ($\pi_j$) of this criterion. We have to stress that the heuristics shown bellow are subjective and could be modified.

### 4.2.1 Default Values of Discriminating Thresholds: ($q_j$) and ($p_j$)

In our opinion, the default value of the indifference threshold should only take into account the data imperfection component. The value of the indifference threshold should always be smaller than the veto and strict preference thresholds. Let $E_j$ is the range of measurement scale associated to criterion *j*. Let the lower limit of the veto threshold be $\Delta.E_j$ if $\pi_j = 1$(we have normalised relative importance coefficients: $\sum_{j=1}^{n} \pi_j = 1$ and $\pi_j \geq 0$). The indifference threshold does not affect and is not affected by the other criteria. So, it is justified to focus on one criterion at a time (set $\pi_j = 1$). $\Delta.E_j$ constitutes an upper bound for $q_j$. We propose the following formula to compute the default value of the indifference threshold:

$$q_j = \beta(I_j)\Delta.E_j, \quad \text{where } \beta(I_j) = \begin{cases} 0.15 & \text{if} \quad (I_j) = L \\ 0.30 & \text{if} \quad (I_j) = M \\ 0.60 & \text{if} \quad (I_j) = H \end{cases} \qquad (18.)$$

Note that the expression of the β function is subjective, and based only on authors' experiences and some simulation experiments. One should try to study other forms of this function and make an extensive simulation tests to figure out the real effect of this function.

The strict preference threshold should have a value somewhere between the veto and the indifference thresholds ($q_j \leq p_j \leq v_j$). Based on the same estimation of the data imperfection level presented before, we suggest the following formula to set the default value of the strict preference threshold for a criterion *j*:

$$p_j = \Delta.E_j + \alpha(I_j)(v_j - q_j), \text{ avec } \quad \alpha(I_j) = \begin{cases} 0.05 & si \quad I_j = L \\ 0.15 & si \quad I_j = M \\ 0.30 & si \quad I_j = H \end{cases} \qquad (19.)$$

Here again, the choice of the α function is purely subjective established by simulation. An extensive study could lead to enhance and justify these functions.

In the context of CASAP, it is easy to set the default values of the indifference and the strict preference thresholds for the criteria $C_1, C_2, C_3$ and $C_{13}$; $E_j \leq 1$ for $j = 1,2,3$ *and* $13$. The range $E_8$ of the criterion $C_8$ scale should be estimated prior to some analysis of many CoAs cost. Then, it is easy to estimate $\Delta E_8'$ : $\Delta E_8' = \max_i(e_{i8}) - \min_i(e_{i8})$. The personnel should afterward make a correction to set a more appropriate value of $E_8$. The range $E_7$ of the criterion $C_7$ scale should be easy to determine: somewhere between 0 and 3.

For distributional criteria (uncertain/fuzzy), one should ignore the component of the thresholds related to data imperfection; it is already modelled by the distribution. Nevertheless, one should remember that these thresholds model also the DM's preferences hesitations [Ref. 40]. We propose to use the thresholds even for distributional criteria. The values assigned to these thresholds should be smaller than those obtained if the criteria are crisp. For example, we suggest dividing the indifference threshold $q_j$ by at least two.

For instance, the default values of the thresholds are supposed to be constant. As we mentioned before, it is useful for some criteria to have variable thresholds function of the position on the measurement scale. This can easily be the case of the criterion $C_8$

(cost of resources). To simplify, we decided to consider all the thresholds as constant. In future work, it will be possible to consider this aspect.

### 4.2.2 Default Values of the Veto Threshold: $(v_j)$

The veto threshold plays a different role from those of the indifference and strict preference thresholds. The veto threshold is considered as an information concerning the inter-criteria. The value for this threshold should be very large. It is reasonable then to neglect the data imperfection component when setting the value of this threshold. We uphold the idea that the value of the veto threshold should only depend on the relative importance of the criteria. In order to set the value of this threshold, we have been inspired from the method suggested by Roy [Ref. 43]. This method begins by normalising the evaluation difference between two CoAs according to the discordant criteria and adjusted using the relative importance of these criteria. This is performed as follows:

$$d_{ikj} = \frac{(e_{ij} - e_{kj}).\pi_j}{E_j} \qquad (20.)$$

In order to build an outranking relation between the two CoAs, the method compares $d_{ikj}$ to a threshold $\Delta$; this threshold plays the role of a veto in the ELECTRE I method. We suggest setting the value of the veto threshold $v_j$ for the selected criteria as follows:

$$e_{ij} - e_{kj} \le \frac{\Delta.E_j}{\pi_j} \mapsto v_j = \frac{\Delta.E_j}{\pi_j} \qquad (21.)$$

A reasonable value of $\Delta$ could be 0.25. However, a simulation analysis shows that for small relative importance coefficients, the computed value of $v_j$ was very large. In order to make a proper correction to this situation, we suggest the following formula:

$$v_j = \frac{\Delta.E_j}{\sqrt{\pi_j}} \quad for\ \pi_j \ge 0.01 \qquad (22.)$$

This relationship between the veto threshold and the criterion relative importance respects the philosophy of the veto introduced originally by Roy [Ref. 32]: the veto threshold is a voluntary parameter with a value decreasing in function of the criterion's relative importance[1].

### 4.3 Balancing the Criteria: Relative Importance Coefficients

Trying to aggregate evaluations obtained on the various criteria requires introducing at one point or another information exterior to performance matrix **E**. We may want to determine the numerical values which we associate with the criteria in accordance with their relative importance. These coefficients of relative importance must be determined with caution, because they are usually confused with weights or substitution rates. Weights or substitution rates represent the ***trade-offs*** the DM is willing to consider between the attributes/criteria. Although the idea of weights or substitution rates is natural, these concepts are not compatible with the role of these coefficients in PAMSSEM. PAMSSEM brings coefficients of intrinsic importance into play. These coefficients represent the *"voting power"* the DM is willing to assign to each criterion. To sum up, the method of determining the coefficients of relative importance is intimately linked to the MCAP to be implemented.

To help the DM to determine the appropriate relative importance coefficient for each criterion, we decided to use a method inspired from "Simos modifiée" [Ref. 44], which is very intuitive for the users. First, the DM has to rank the criteria from the most important to the less important:

$$C_{(1)}, C_{(2)}, \ldots, C_{(n)} \tag{23.}$$

Where $C_{(1)}$ is the most important criterion, $C_{(n)}$ is the less important criterion, and $n$ is the number of criteria to be ranked. It is assumed that any criterion that is not in that ranking will be considered to have 0 as relative importance coefficient (which can be considered with almost no effect on the ranking of CoAs), and any criteria in the same level would have the same importance (*ex æquo*).

---

[1] Free translation of *"le seuil de veto est une donnée volontariste dont la valeur est, en règle générale, d'autant plus petite que le*

Basic units are used to help represent the difference of importance between the different criteria. A basic unit represents the minimal interval difference that the DM wants to consider when saying that a criteria is more important than another one. The DM has to put as many graphic basic units as needed between each criterion to represent the difference of importance that the DM thinks there is between them (see Figure 7).

Once done, it is possible to assign a level for each criterion. This level can be determined as follows:

- The level of $C_{(1)}$ corresponds to the level one :

$$L_{C(n)} = 1 \tag{24.}$$

- The level of each other criterion corresponds to :

$$L_{C(i)} = L_{C(i+1)} + \Delta U_{C(i),C(i+1)} + 1 \text{ , } \forall i = 1,..,n-1 \tag{25.}$$

where $\Delta U_{C(i),C(i+1)}$ is the number of basic units between $C_{(i)}$ and $C_{(i+1)}$.

The DM has to determine how many times $C_{(1)}$ is more important than $C_{(n)}$; this value is called $\Delta_{\text{importance}}$. Considering that the relative importance of $C_{(n)}$ is one and the one of $C_{(1)}$ is $\Delta_{\text{importance}}$, it is possible to calculate the relative importance of the other criteria by solving this simple system of equations:

$$aL_{C(n)} + b = 1 \tag{26.}$$

$$aL_{C(1)} + b = \Delta_{\text{importance}} \tag{27.}$$

Then the relative importance of each criterion ($RW_{C(i)}$) can be calculated by:

$$RW_{C(i)} = aL_{C(i)} + b \text{ , } \forall i = 1,..,n \tag{28.}$$

where $a = \left( \dfrac{\Delta_{\text{importance}} - 1}{L_{C(1)} - 1} \right)$, and $b = \left( \dfrac{L_{C(1)} - \Delta_{\text{importance}}}{L_{C(1)} - 1} \right)$.

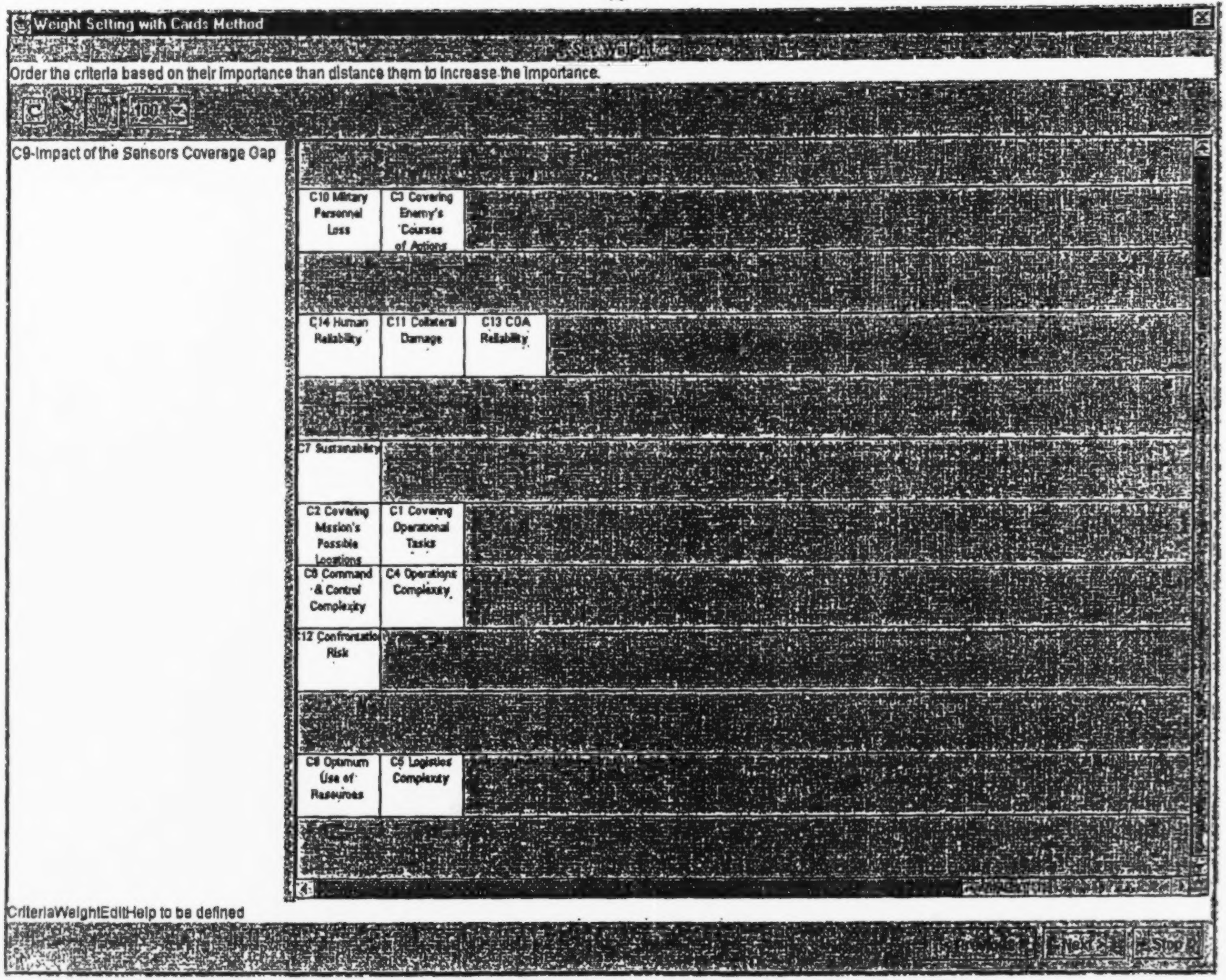
---

*poids est grand*".

The normalised relative importance coefficient for each criterion ($NW_{C(i)}$) will be given as follows:

$$NW_{C(i)} = \frac{RW_{C(i)}}{\sum_{j=1}^{n} RW_{C(j)}}, \ \forall i = 1,..,n \qquad (29.)$$

An example of a result of the criteria balancing method is shown by Figure 8.



Weight Setting with Cards Method

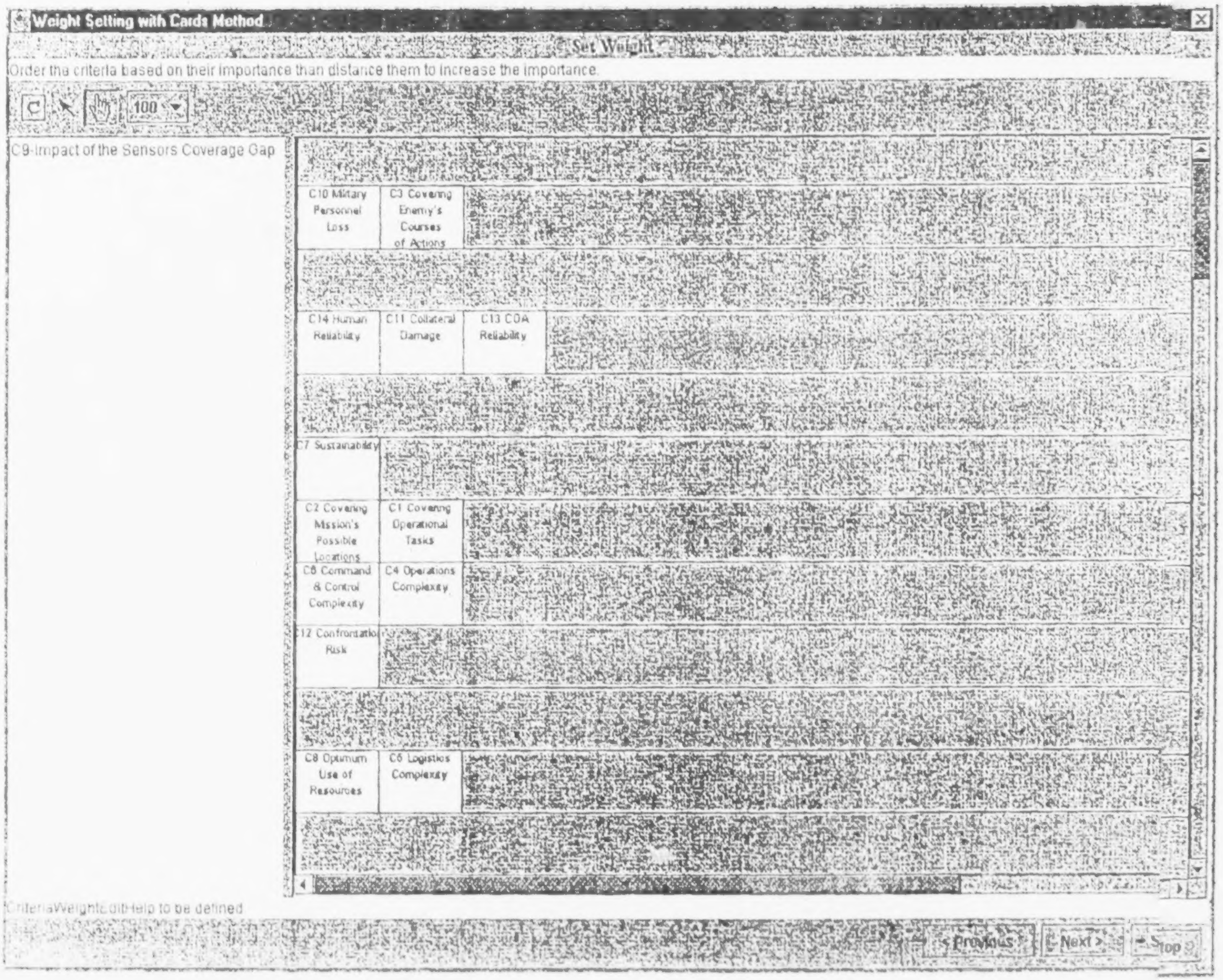
Order the criteria based on their importance than distance them to increase the importance.

C9-Impact of the Sensors Coverage Gap

CriteriaWeightEditHelp to be defined

Set Weight
100
C10 Military Personnel Loss
C3 Covering Enemy's Courses of Actions
C14 Human Reliability
C11 Collateral Damage
C13 COA Reliability
C7 Sustainability
C2 Covering Mission's Possible Locations
C1 Covering Operational Tasks
C6 Command & Control Complexity
C4 Operations Complexity
C12 Confrontation Risk
C8 Optimum Use of Resources
C5 Logistics Complexity
< Previous
Next >
Stop

FIGURE 7 - Illustration of the criteria balancing method implemented in CASAP



Weight Setting with Cards Method

Set Weight

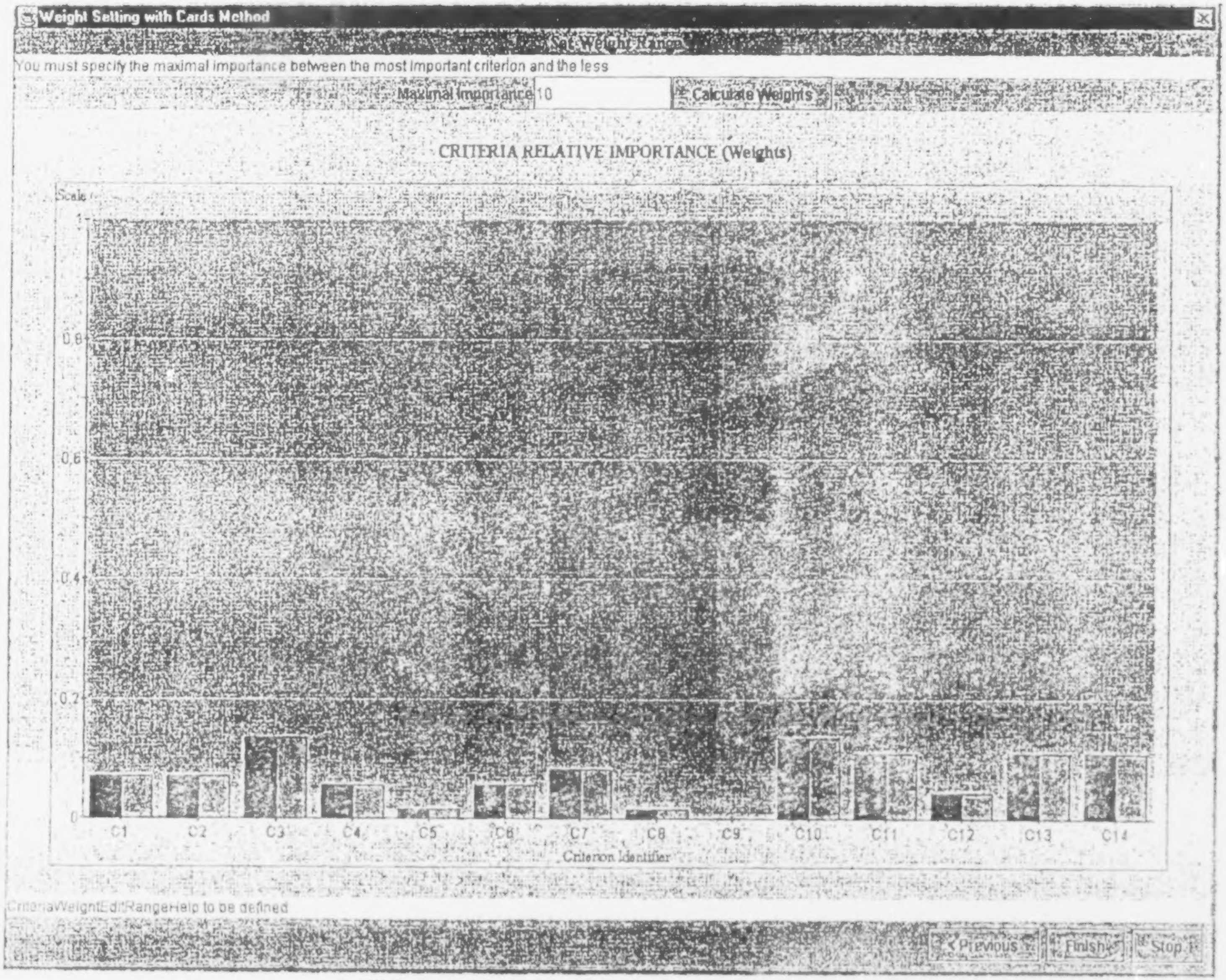
Order the criteria based on their importance than distance them to increase the importance.

C9-Impact of the Sensors Coverage Gap

100
C10 Military Personnel Loss
C3 Covering Enemy's Courses of Actions
C14 Human Reliability
C11 Collateral Damage
C13 COA Reliability
C7 Sustainability
C2 Covering Mission's Possible Locations
C1 Covering Operational Tasks
C6 Command & Control Complexity
C4 Operations Complexity
C12 Confrontation Risk
C8 Optimum Use of Resources
C5 Logistics Complexity
< Previous
Next >
Stop

CriteriaWeightEditHelp to be defined

FIGURE 7 - Illustration of the criteria balancing method implemented in CASAP



Weight Setting with Cards Method
Set Weight Range

You must specify the maximal importance between the most important criterion and the less

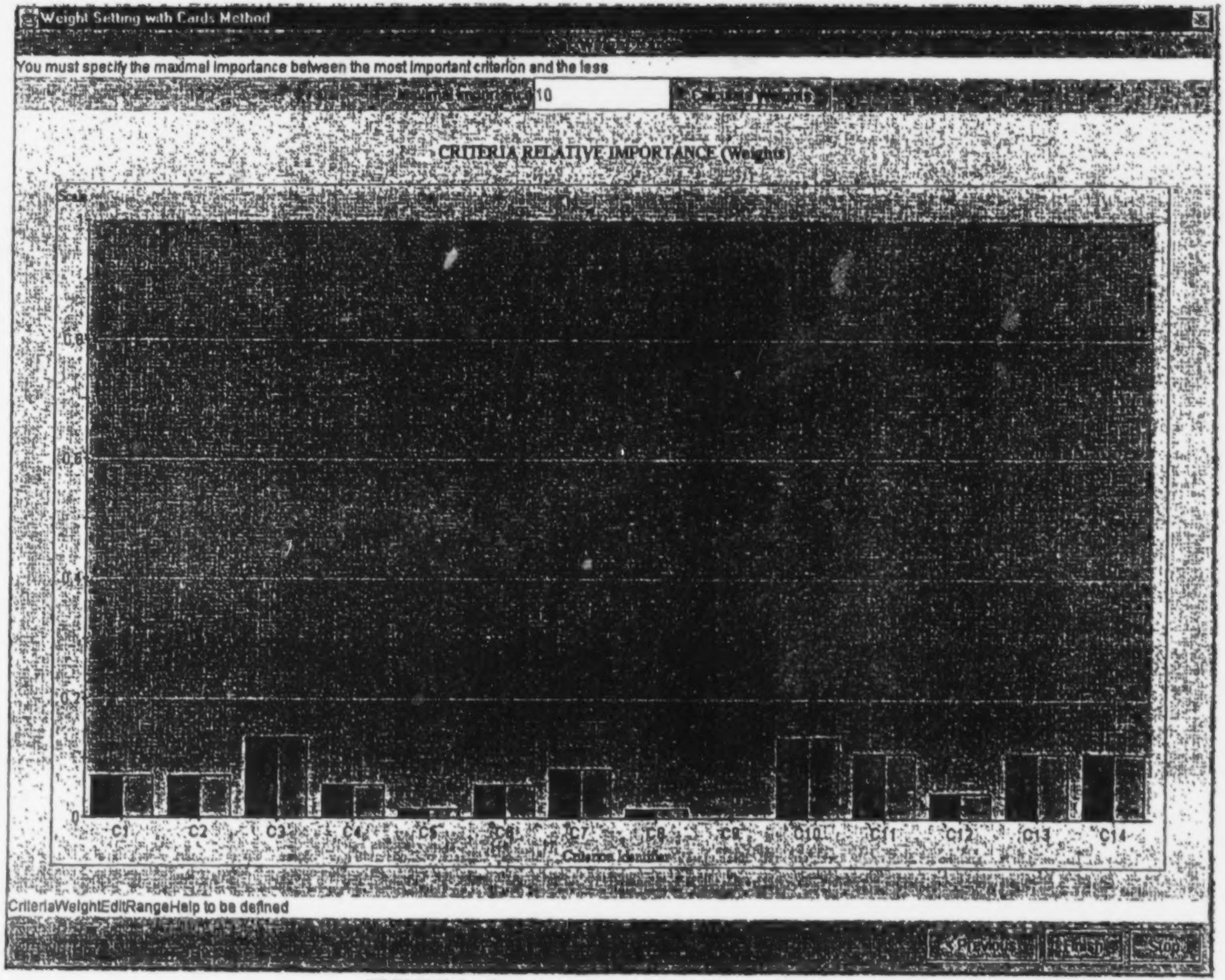
Maximal importance 10

Calculate Weights
CRITERIA RELATIVE IMPORTANCE (Weights)
Scale
1
0.8
0.6
0.4
0.2
0
C1
C2
C3
C4
C5
C6
C7
C8
C9
C10
C11
C12
C13
C14
Criterion Identifier

CriteriaWeightEditRangeHelp to be defined

< Previous
Finish
Stop

FIGURE 8 - Result of the criteria balancing method



Weight Setting with Cards Method

You must specify the maximal importance between the most important criterion and the less

Maximal importance: 10

CriteriaWeightEditRangeHelp to be defined

Calculate Weights
CRITERIA RELATIVE IMPORTANCE (Weights)
Scale
1
0.8
0.6
0.4
0.2
0
C1
C2
C3
C4
C5
C6
C7
C8
C9
C10
C11
C12
C13
C14
Criterion Identifier
< Previous
Finish
Stop

FIGURE 8 - Result of the criteria balancing method

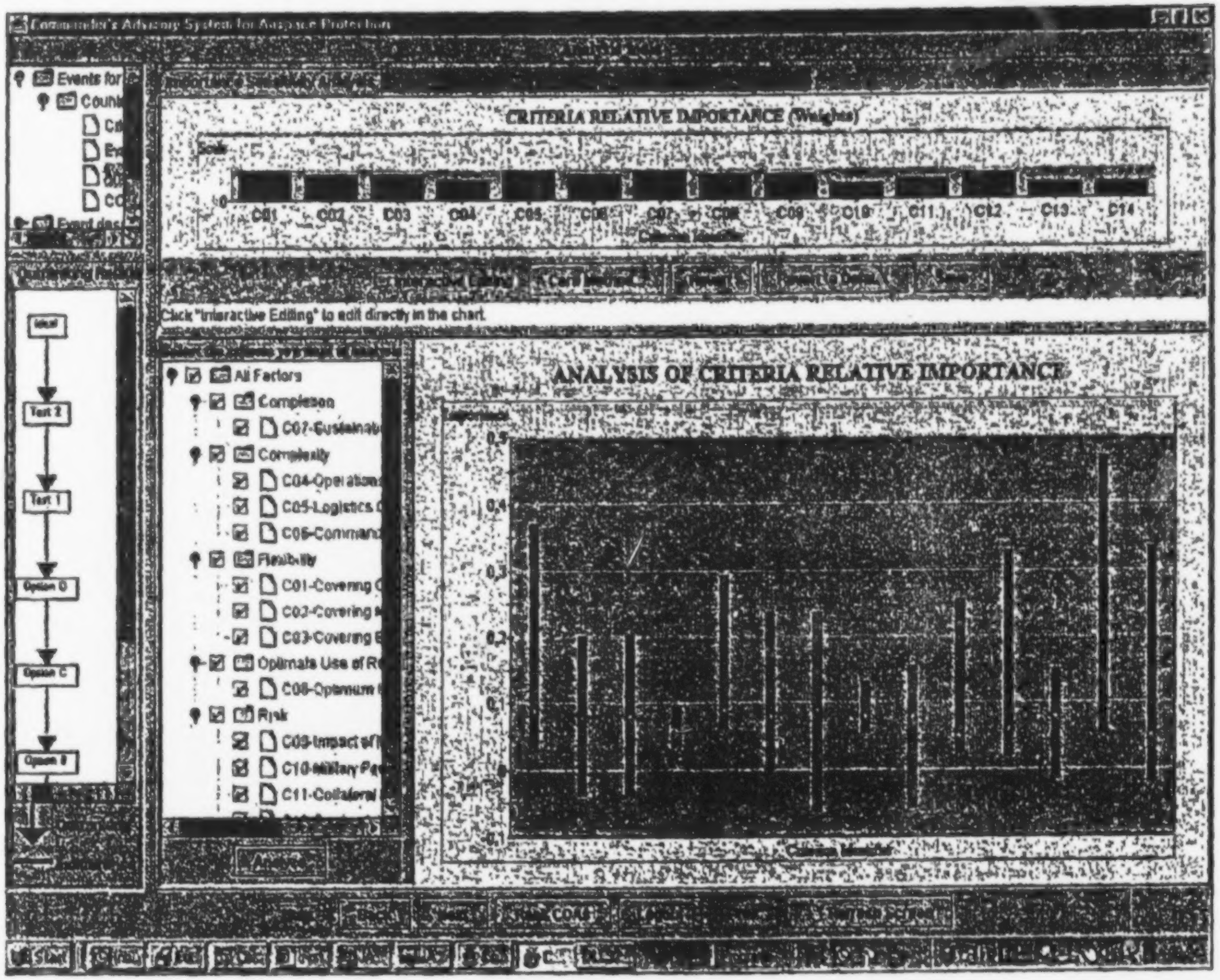
# 5.0 ANALYSIS TOOLS FOR PAMSSEM II

## 5.1 Relative Importance Coefficients Stability Analysis

The relative importance coefficients of the criteria play a major role in the outcome of the evaluation and aggregation process. These coefficients are in reality estimates of the importance that the decision maker (Commander) gives to each criterion in order to balance a decision. This process involves imprecise and vague judgements. The values $\pi_j$ given to the relative importance coefficients of the different criteria $j$ are not very precise. In these circumstances, it is helpful to determine to what extent the solution obtained is sensitive to the relative importance coefficient variations. In this section, we introduce the sensitivity analysis for PAMSSEM. This analysis leads to determine stability intervals for the relative importance coefficient of each criterion $\pi'_j$. These intervals consist of the values that the relative importance coefficient of each criterion can take without any order reversal. This analysis can help the Commander (or the person in charge) of the AOC identifying the sensitive issues that can affect a decision. The mathematical formulation for the analysis will be presented in another report.

In the context of the CASAP, the Commander (or the person in charge of the AOC) can use the sensitivity analysis at any time following the evaluation process. As shown in Figure 9, in the selection window, the Commander is asked to select the criteria for which he/she wants to perform the analysis. He/she has the possibility to select all the criteria or a sub-set. In the second case, he/she can either select all the criteria related to specific factor by selecting the appropriate factor, or select directly particular criteria. Afterwards, by activating the button "*Analyze*", he/she obtains a chart of the relative importance coefficients stability intervals for each criterion selected (Figure 9).

In order to implement the sensitivity analysis, we used an algorithm of the simplex method. The computation performances are very good. The experiments performed show that the results are displayed in a fraction of a second after the analysis is activated. Many intermediate results are retrieved from the first PAMSSEM II computations. These results are used to compute the different parameters of the mathematical programme (45). The number of constraints of this programme is limited at most to $(m+6n)$ with only $2n$ variables.


Commander's Advisory System for Airspace Protection
Events for
Count
CRITERIA RELATIVE IMPORTANCE (Weights)
0
C01
C02
C03
C04
C05
C06
C07
C08
C09
C10
C11
C12
C13
C14
Interactive Editing
All Factors
Complexion
C07-Sustainab
Complexity
C04-Operations
C05-Logistics
C06-Command
Flexibility
C01-Covering
C02-Covering
C03-Covering
Optimale Use of R
C08-Optimum
Risk
C09-Impact of
C10-Military P
C11-Collateral
ANALYSIS OF CRITERIA RELATIVE IMPORTANCE
0.5
0.4
0.3
0.2
0.1
0
-0.1
Ideal
Test 2
Test 1
Option D
Option C
Option B
Start

Click "Interactive Editing" to edit directly in the chart.

FIGURE 9 - Example of the results of the relative importance coefficient stability analysis

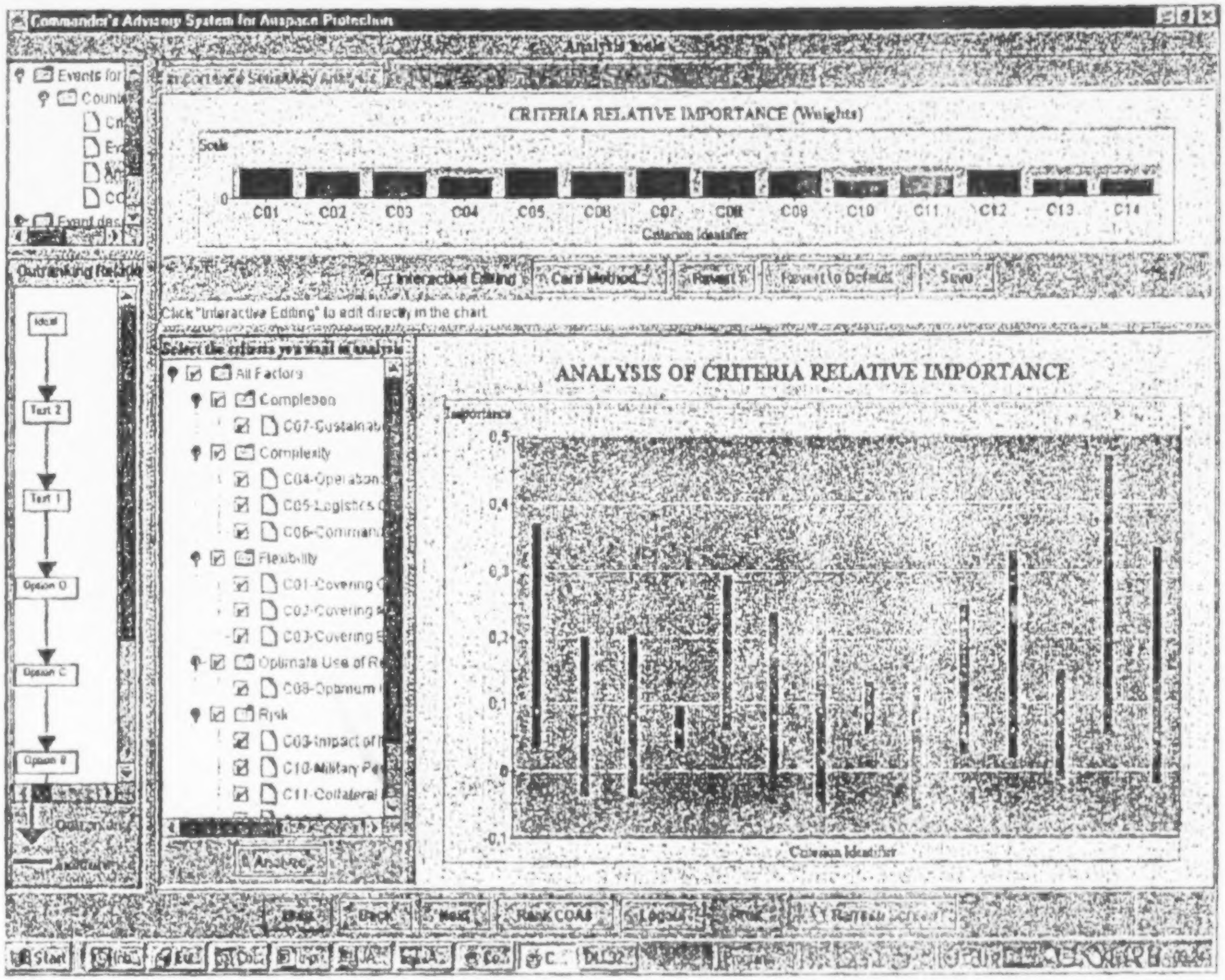

Commander's Advisory System for Airspace Protection
Analysis tools
Importance Sensitivity Analysis
Events for
Countr
Event des
CRITERIA RELATIVE IMPORTANCE (Weights)
Scale
0
C01
C02
C03
C04
C05
C06
C07
C08
C09
C10
C11
C12
C13
C14
Criterion Identifier
Interactive Editing
Card Method
Revert
Revert to Default
Save
Click "Interactive Editing" to edit directly in the chart
Outranking Relatio
Ideal
Test 2
Test 1
Option D
Option C
Option B
Select the criteria you want to analyze
All Factors
Completion
C07-Sustainab
Complexity
C04-Operation
C05-Logistics
C06-Comman
Flexibility
C01-Covering
C02-Covering
C03-Covering
Optimal Use of R
C08-Optimum
Risk
C09-Impact of
C10-Military Pe
C11-Collateral
Analyze
ANALYSIS OF CRITERIA RELATIVE IMPORTANCE
Importance
0,5
0,4
0,3
0,2
0,1
0
-0,1
Criterion Identifier
Back
Next
Rank COAs
Legend
Print
Refresh Screen
Start

FIGURE 9 - Example of the results of the relative importance coefficient stability analysis

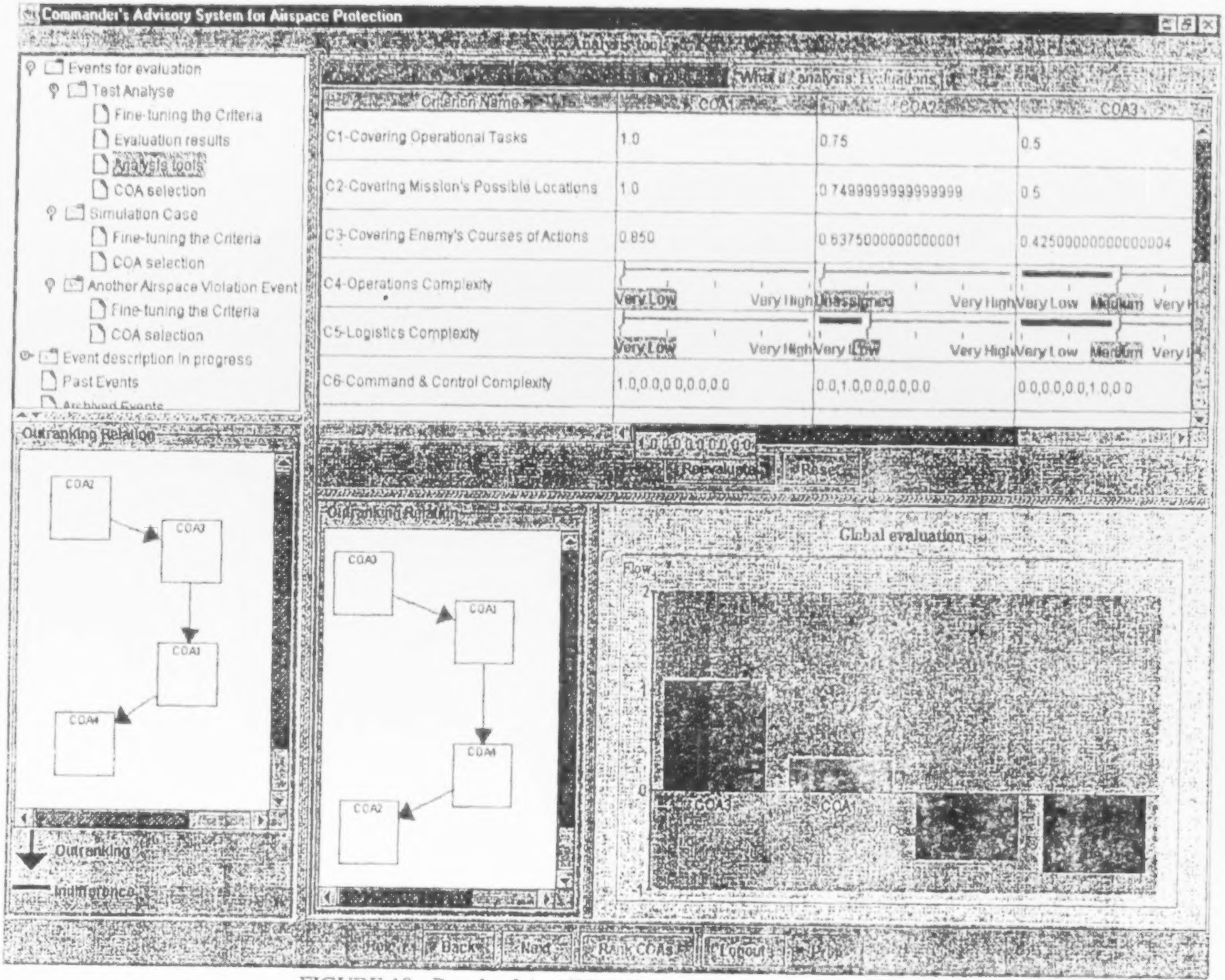
## 5.2 "What if Analysis"

The evaluation of the CoAs according to the different criteria is obtained by combining information introduced by the operators. This information can be uncertain, ambiguous, fuzzy, and subjective. Moreover, some evaluations could be missing or impossible to obtain in real time. For all these reasons, it is appropriate to consider the data used for the evaluation as imprecise and subjective. In this situation, the Commander needs to verify the effects of any modification in the evaluations of one or more CoA on the outcomes of the aggregation process. We thought it is useful to develop and implement heuristics in order to make "what if" analyses. These analyses are for instance limited to the CoAs evaluations (Multiple Criteria table) and to PAMSSEM's parameters (thresholds).

### 5.2.1 "What if Analysis" on the Evaluations

Some of the CoAs evaluations could be subjective, uncertain or even subject to condition variations (ex. weather). In this case, the "what if analysis" on the CoAs evaluations consists of answers to the following question: *"What could happen to the actual ranking if one or more evaluations of one or many CoAs change?"* This type of question may lead to recognise possible weaknesses and strengths of particular CoAs. Moreover, if some CoAs are globally very close one to another. In this situation the Commander will have some doubts about a clear choice between these CoAs. But considering the subjectivity and the uncertainties of the evaluation processes, one should verify more deeply these ties. The "what if" analysis on the evaluations may help the Commander to discriminate between the tied CoAs. Also, if the Commander does not agree with some evaluation given to a particular CoA, he/she can use this tool to modify such evaluation and seek the result. In this way, he/she may gain the time to go again through the evaluation process. We think this tool is powerful and help fulfilling the role of decision aid devoted to CASAP.

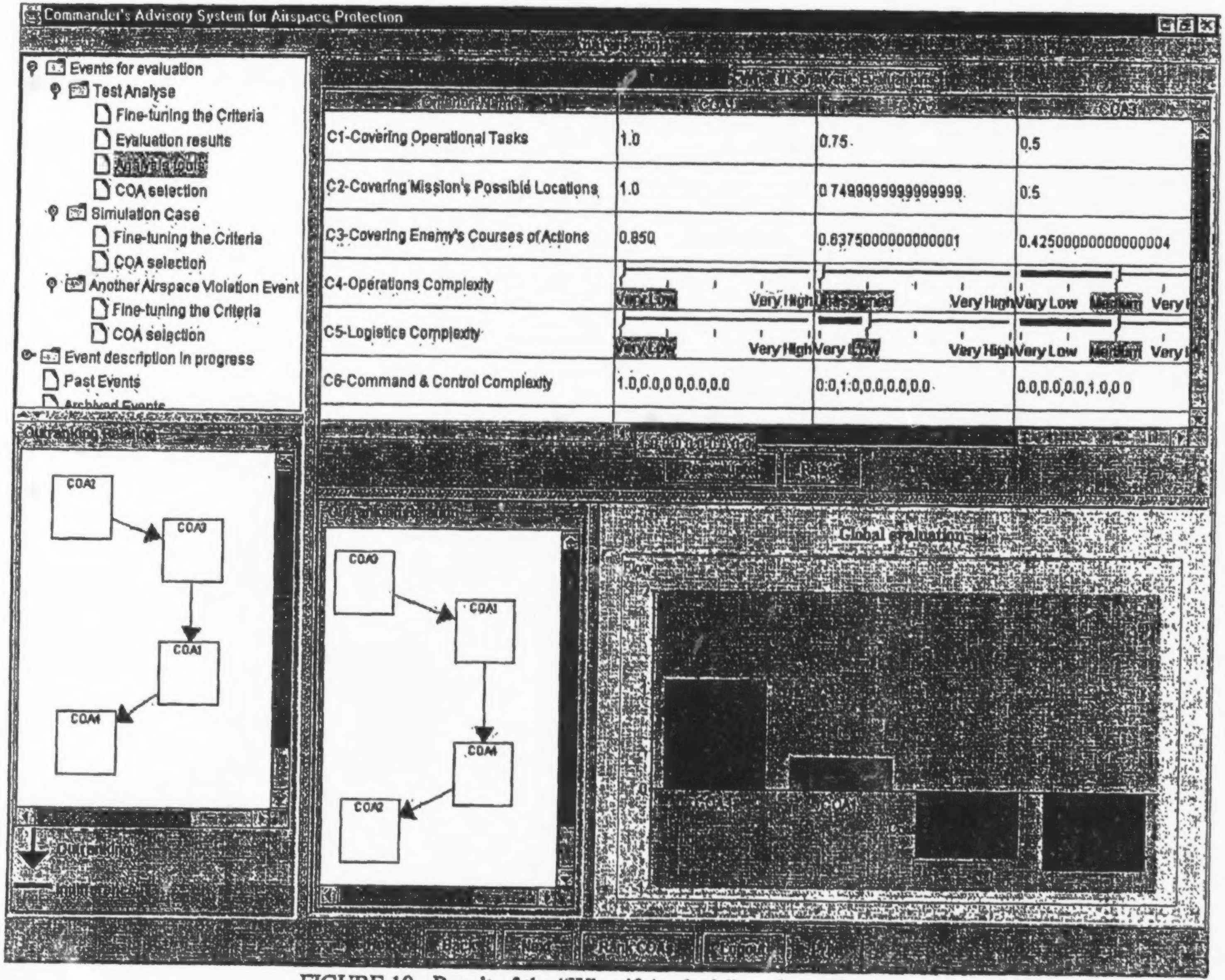
The way we chose to implement this analysis is based on the representation of the Multiple Criteria table **E**. **E** syntheses the evaluations of the different CoAs according to the different criteria. The user (Commander) can change online any evaluation. Then, by activating the *"Analyze"* button, he/she obtains the new results considering the modifications. The user may perform this analysis as often as wishes. The new result is displayed in contrast with the initial one obtained by the original evaluations (see Fig. 10). The analysis of the two results may lead the user to identify critical factors or risky issues for his decision. Moreover, he/she may identify ways to improve the proposed CoAs. We believe that this analysis tool is powerful to help Commander justify his decisions.


Commander's Advisory System for Airspace Protection
Analysis tools
What if analysis Evaluations
Events for evaluation
Test Analyse
Fine-tuning the Criteria
Evaluation results
Analysis tools
COA selection
Simulation Case
Fine-tuning the Criteria
COA selection
Another Airspace Violation Event
Fine-tuning the Criteria
COA selection
Event description in progress
Past Events
Archived Events
Outranking Relation
COA2
COA3
COA1
COA4
Outranking
Indifference
Reevaluate
Reset
Global evaluation
Flow
2
1
0
-1
Help
Back
Next
Rank COAs
Logout

| Criterion Name | COA1 | COA2 | COA3 |
|---|---|---|---|
| C1-Covering Operational Tasks | 1.0 | 0.75 | 0.5 |
| C2-Covering Mission's Possible Locations | 1.0 | 0.7499999999999999 | 0.5 |
| C3-Covering Enemy's Courses of Actions | 0.850 | 0.6375000000000001 | 0.4250000000000004 |
| C4-Operations Complexity | Very Low … Very High | Unassigned … Very High | Very Low Medium Very H |
| C5-Logistics Complexity | Very Low … Very High | Very Low … Very High | Very Low Medium Very H |
| C6-Command & Control Complexity | 1.0,0.0,0.0,0.0,0.0 | 0.0,1.0,0.0,0.0,0.0 | 0.0,0.0,0.0,1.0,0.0 |

1.0,0.0,0.0,0.0,0.0

FIGURE 10. Result of the "What if Analysis" on the evaluations


Commander's Advisory System for Airspace Protection
Events for evaluation
Test Analyse
Fine-tuning the Criteria
Evaluation results
Analysis tools
COA selection
Simulation Case
Fine-tuning the Criteria
COA selection
Another Airspace Violation Event
Fine-tuning the Criteria
COA selection
Event description in progress
Past Events
Outranking relation
COA2
COA3
COA1
COA4
Outranking
Indifference
COA0
COA1
COA4
COA2
Global evaluation
Flow
COA1
Back
Next
Rank COAs
Cancel

| Criterion Name | [illegible] | COA2 | COA3 |
|---|---|---|---|
| C1-Covering Operational Tasks | 1.0 | 0.75 | 0.5 |
| C2-Covering Mission's Possible Locations | 1.0 | 0.7499999999999999 | 0.5 |
| C3-Covering Enemy's Courses of Actions | 0.850 | 0.6375000000000001 | 0.42500000000000004 |
| C4-Operations Complexity | Very Low Very High | Medium Very High | Very Low Medium Very H |
| C5-Logistics Complexity | Very Low Very High | Very Low Very High | Very Low Medium Very H |
| C6-Command & Control Complexity | 1.0,0.0,0 0,0.0,0.0 | 0.0,1.0,0.0,0.0,0.0 | 0.0,0.0,0.0,1.0,0 0 |

FIGURE 10. Result of the "What if Analysis" on the evaluations

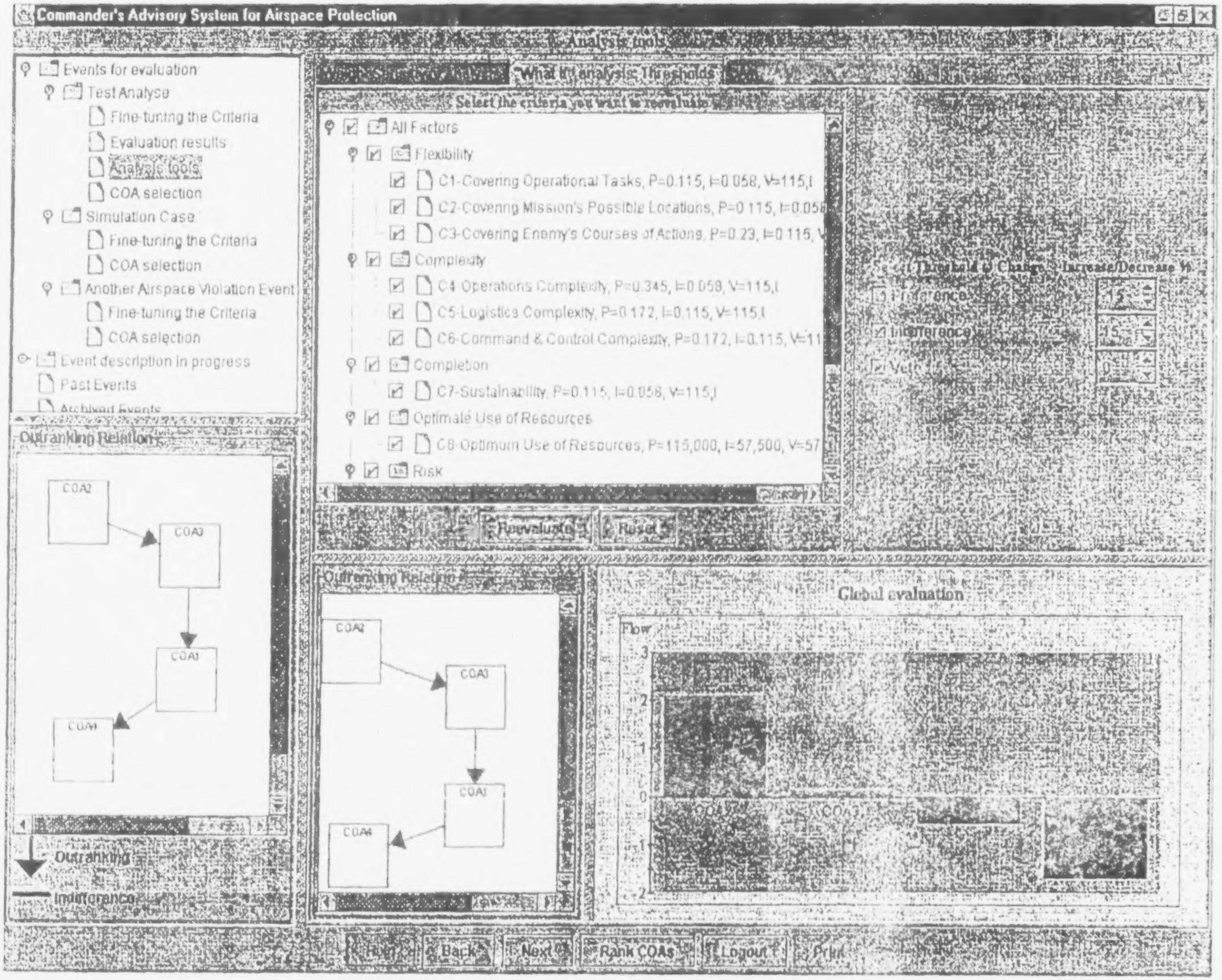
### 5.2.2 "What if Analysis" on the Tresholds

PAMSSEM uses thresholds to model the DM's preference structure. According to each criterion, the DM may have one or another of the following attitudes when asked to compare two CoAs:

- indifferent between the two: indifference situation,
- clearly prefers one: preference situation,
- hesitation between the indifference and the preference situations: weak preference situation.

In order to consider these situations, we introduced two thresholds for each criterion: indifference and preference thresholds. Moreover, some criteria may be considered as so important that they may have special considerations. The veto threshold was then introduced in order to oppose the selection of a CoA if it is relatively weak on one of these criteria.

An analysis tool was designed to help the user figure out what role these parameters may play in the outcome of PAMSSEM. The user may choose to perform the *"What if analysis"* on one or all the thresholds for a sub-set or all the criteria. The user selects for which criteria he/she wants to perform the analysis. He/she selects what kind of thresholds to consider in this analysis, and fix a percentage of variation of each kind (see Fig. 11). By activating the button "Analyze", the system displays the new result in contrast with the one obtained by the original values of the thresholds. The user may repeat this analysis as often as needed. The result of this analysis is very useful to help the DM analyse his preferences and discover his aversion to risk, for example.



Commander's Advisory System for Airspace Protection
Analysis tools
What if analysis: Thresholds

- Events for evaluation
  - Test Analyse
    - Fine-tuning the Criteria
    - Evaluation results
    - Analysis tools
    - COA selection
  - Simulation Case
    - Fine-tuning the Criteria
    - COA selection
  - Another Airspace Violation Event
    - Fine-tuning the Criteria
    - COA selection
- Event description in progress
- Past Events
- Archived Events

Select the criteria you want to reevaluate

- ☑ All Factors
  - ☑ Flexibility
    - ☑ C1-Covering Operational Tasks, P=0.115, I=0.058, V=115,I
    - ☑ C2-Covering Mission's Possible Locations, P=0.115, I=0.058
    - ☑ C3-Covering Enemy's Courses of Actions, P=0.23, I=0.115, V
  - ☑ Complexity
    - ☑ C4-Operations Complexity, P=0.345, I=0.058, V=115,I
    - ☑ C5-Logistics Complexity, P=0.172, I=0.115, V=115,I
    - ☑ C6-Command & Control Complexity, P=0.172, I=0.115, V=11
  - ☑ Completion
    - ☑ C7-Sustainability, P=0.115, I=0.058, V=115,I
  - ☑ Optimale Use of Resources
    - ☑ C8-Optimum Use of Resources, P=115,000, I=57,500, V=57
  - ☑ Risk

Threshold % Change Increase/Decrease %

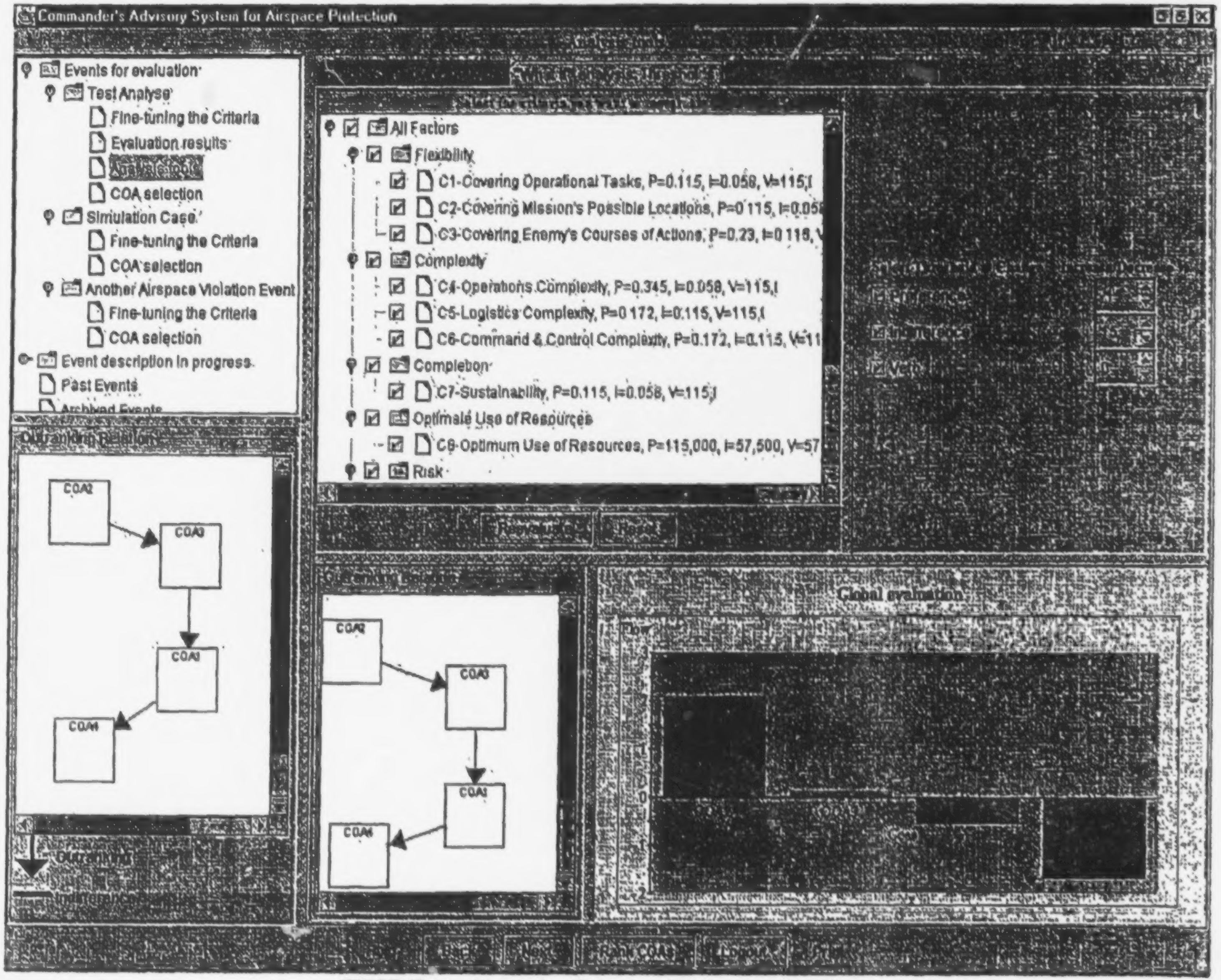
☑ Preference 12

☑ Indifference 15

☑ Veto 0

Reevaluate
Reset
Outranking Relation
COA2
COA3
COA1
COA4
Outranking
Indifference
Outranking Relation
COA2
COA3
COA1
COA4
Global evaluation
Flow
3
2
1
0
-1
-2
COA
COA
Help
Back
Next
Rank COAs
Logout
Print

FIGURE 11 – Example of the thresholds what if analysis


Commander's Advisory System for Airspace Protection
Outranking Relations
COA2
COA3
COA1
COA4
Global evaluation
Reevaluate
Reset

- Events for evaluation
  - Test Analyse
    - Fine-tuning the Criteria
    - Evaluation results
    - Analysis tools
    - COA selection
  - Simulation Case
    - Fine-tuning the Criteria
    - COA selection
  - Another Airspace Violation Event
    - Fine-tuning the Criteria
    - COA selection
- Event description in progress.
- Past Events
- Archived Events

- ☑ All Factors
  - ☑ Flexibility
    - ☑ C1-Covering Operational Tasks, P=0.115, I=0.058, V=115,I
    - ☑ C2-Covering Mission's Possible Locations, P=0.115, I=0.05
    - ☑ C3-Covering Enemy's Courses of Actions, P=0.23, I=0.118, V
  - ☑ Complexity
    - ☑ C4-Operations Complexity, P=0.345, I=0.058, V=115,I
    - ☑ C5-Logistics Complexity, P=0.172, I=0.115, V=115,I
    - ☑ C6-Command & Control Complexity, P=0.172, I=0.115, V=11
  - ☑ Completion
    - ☑ C7-Sustainability, P=0.115, I=0.058, V=115,I
  - ☑ Optimale Use of Resources
    - ☑ C8-Optimum Use of Resources, P=115,000, I=57,500, V=57
  - ☑ Risk

FIGURE 11 – Example of the thresholds what if analysis

## 6.0 DISCUSSION AND FURTHER DEVELOPMENTS

The majority of the decision-aid prototypes do not give enough attention to the presentation and explanation of the results obtained by applying one or another decision-analysis model, including an MCAP. If we target to minimise the "black box" impression, one should arm CASAP with result presentation and explanation facilities. The first version of CASAP will include many display facilities and analysis tools. However, the explanations are not satisfactory. In fact, developing an explanation facility within a decision support system appears to us to be very challenging task. The explanations should be provided in function of the DM needs, level of comprehension, cognition, behaviour, etc.

On another front, the data obtained from the DM and the AOC staff (operators) is often imperfect and subjective due to the nature of DMS they are confronting and the difficulties of DMP. Therefore, it often happens that the set of data is not unique for representing the situation; several plausible sets of data may possibly exist [Ref. 45]. For example, a scenario may present a particular set of values for CoAs evaluations according to chosen criteria, and a particular setting of PAMSSEM parameters. PAMSSEM should identify the best compromise among the considered possible CoAs for this scenario. However, we are still not sure that PAMSSEM would produce the same ranking for another scenario (another set of values) representing the same situation. In this case, we suggest to add to CASAP a useful tool to search for *robust results* (compromise, recommendation,...).

A pairwise comparison is a powerful tool to analyse and prioritise different CoAs according to different criteria. In fact, this kind of comparison is performed according to the same criterion each time, which means to use the same measurement scale and it is always possible to establish either an indifference, weak or a strict preference between the two alternatives. Then the aggregation is performed on the results of the pairwise comparisons. This solves the problems of incommensurability and conflicting criteria. Although the pairwise comparison is a powerful tool, it hides a difficulty: the relativity of the comparison process[1]. In order to overcome this difficulty jeopardising the implementation of PAMSSEM, we suggest to add other tools e.g. to help screen the CoAs.

## 6.1 Results Displays Facilities

The results displays facilities should include visual displays to clearly show the global as well as intermediate results to the DM. At the global level, one should consider showing the following elements:

- a graph to illustrate the partial and the total preorders obtained respectively by PAMSSEM I and PAMSSEM II;
- the possible incomparabilities among some CoAs;
- a graphical display of the entering, leaving and net flows (histograms, charts, ...);
- a graphical display for the analysis tools.

Intermediate results could be displayed in different manners. For example, CASAP could show, at the convenience of the users, displays of the following information:

- the performance matrix **E** and the settings of PAMSSEM's parameters;
- relative positions of the CoAs according to each criterion;
- the discordance and concordance matrixes;
- the outranking matrix, and an extensive explanation of the outranking relationship between each pair of CoAs;
- a graphical display of the outranking relationships of the CoAs relative to a targeted CoA;

---

[1] Comparing two CoAs one to another; there are no static reference.

- a graphical display of the weakness ($\sigma^-(a_i,a_k)$) and strengths ($\sigma^+(a_i,a_k)$) of the CoAs;
- a multidimensional graphical display of the CoAs according the criteria and an identification of the decision axis (taking into consideration the conflicting aspects and the relative importance of the criteria).

For this latest element (multidimensional graphical display), one could build a Geometrical Analysis for Interactive Aid (GAIA) plan as it was performed for PROMETHEE II [Ref. 46]; performing an analytical decomposition of the net flow. A principal components analysis leads to obtain this plan. It is then necessary to compute a $\Phi$ matrix of all the criteria flows. This matrix is harder to obtain for PAMSSEM than it is for PROMETHEE; PAMSSEM computes the outranking degrees ($\sigma(a_i,a_k)$) using a concordance and discordance analyses, which is not the case of PROMETHEE. Let us recall the formulation of the outranking degree:

$$\begin{aligned}\sigma(a_i,a_k) &= C(a_i,a_k)\cdot\left[\prod_{j=1}^{n}\left(1-D_{j,j}^{3}(a_i,a_k)\right)\right]\\ &= \sum_{j=1}^{n}\left[\pi_j\cdot\delta_j(a_i,a_k)\right]\cdot X_{ik}\end{aligned} \tag{30.}$$

As we argued in Chapter 5, $X_{ik}$ is not considered in PROMETHEE. PAMSSEM computes the net flow for each CoA $\Phi(a_i)$:

$$\Phi(a_i)=\sum_{j=1}^{n}\pi_j\cdot\Phi_j(a_i) \tag{31.}$$

where

$$\Phi_j(a_i)=\sum_{k=1,k\neq i}^{m}\left[(\delta_j(a_i,a_k)\cdot X_{ik}-\delta_j(a_k,a_i)\cdot X_{ki}\right] \tag{32.}$$

For the sake of the presentation, it is preferable to consider the normalised flows. Each CoA is compared with $m$-1 other CoAs, then we can define a normalised flow as:

$$\varphi(a_i)=\frac{\Phi(a_i)}{m-1}=\sum_{j=1}^{n}\pi_j\cdot\varphi_j(a_i) \qquad (33.)$$

where

$$\varphi_j(a_i)=\frac{1}{m-1}\sum_{k=1,k\neq i}^{m}\left[(\delta_j(a_i,a_k)\cdot X_{ik}-\delta_j(a_k,a_i)\cdot X_{ki}\right] \qquad (34.)$$

For each CoA $a_i$, $\varphi_j(a_i)$ is only depending on the criterion $j$ ($X_{ik}$ is a correction factor computed according to all the criteria), and is called a *Single-Criterion Associated Flow*. Each CoA will be then characterised by $n$ associated flows and could be represented as a point in an $\Re^n$ space; each dimension corresponds to each criterion. In general, the number of criteria is greater than 2; the DMS in the context of CAP considers 14 criteria. In this situation it is difficult to obtain a direct visual view of the CoAs distribution in an $n$ dimensional space. The GAIA plan consists of an orthogonal projection from the $\Re^n$ space to the $\Re^2$ space using the principal components analysis. For example, in this plan, a criterion's axis length indicates the role of this criterion to discriminate the CoAs. Two criteria with opposite direction axes are completely conflictual.

## 6.2 Robustness

In a decision-aid context, the data are particular representations of the reality, or the result of subjective/imprecise judgements. The concept of robustness can be associated to the plausible scenarios. If all the evaluations, and parameters settings are really objective entities, then there will be a single scenario of data. In this case, we will obtain not only the best compromise, but also a robust compromise. However, if we consider the data imperfections, then we should deal with many scenarios. A robust compromise will be seen in this context as "*a good*" result whichever the scenario a same situation. A scenario for PAMSSEM encloses the evaluations contained in the performance matrix **E**, the relative importance coefficients of the criteria ($\pi_j$), and the discriminating thresholds ($q_j(e_{ij})$, $p_j(e_{ij})$) and veto ($v_j(e_{ij})$). Since PAMSSEM deals with the evaluations imperfections (uncertainties, fuzzy and linguistic evaluations, missing,...), then we can assume that the result is the "best" compromise for any table **E**. A scenario will concern the different modelling parameters introduced by PAMSSEM: $\pi_j$, $q_j(e_{ij})$, $p_j(e_{ij})$, and $v_j(e_{ij})$. In this case, the combinatory of plausible scenarios will be reduced significantly. From the plausible scenarios it is possible to identify a partition (a representative sub-set) to represent all possibilities. A robust result could be the best compromise among all the results obtained by considering each time a scenario of the partition. PAMSSEM will produce a ranking according to each scenario of the partition. Then, it will be possible, using an algorithm, to aggregate the different rankings (as proposed by Ben Khélifa and Martel [Ref. 47]; Xu *et al.* [Ref. 48]) to obtain a robust ranking. It is not the only way to obtain such robust preorder using PAMSSEM. For example it is possible to perform a series of global sensitivity analyses by considering one scenario at a time. Then it is possible to deduce a robust partial ranking from the intersection between the different results.

## 6.3 Screening the CoAs

As we mentioned, the pairwise comparison is a powerful tool when comparing a set of CoAs evaluated according to incommensurable and conflicting criteria. Moreover, it is appropriate to model the DM's preferences according to each criterion, then perform the aggregation. In fact, comparing two CoAs according to the same criterion is not very demanding. Moreover, the cognitive burden to articulate its preferences according to one dimension is always low. However, we should recall that the relative comparison might hide the problem of comparing weak alternatives: selecting the best among the worst. The quality of the best CoA will depend on the quality of the set of CoAs considered. In CASAP, we suggest to introduce a screening procedure before applying PAMSSEM to screen the CoAs. This procedure consists in fixing passing levels on all, or a sub-set, the criteria (Figure 12). Each time a new CoA is introduced, CASAP verifies if it achieves the minimum requirement according to each criterion. This is a quality insurance procedure for the set of CoAs. A dominance check might help to identify the dominant CoA. CASAP includes also a dominance check procedure, which verifies if there is a CoA better than all the other according to all the criteria.

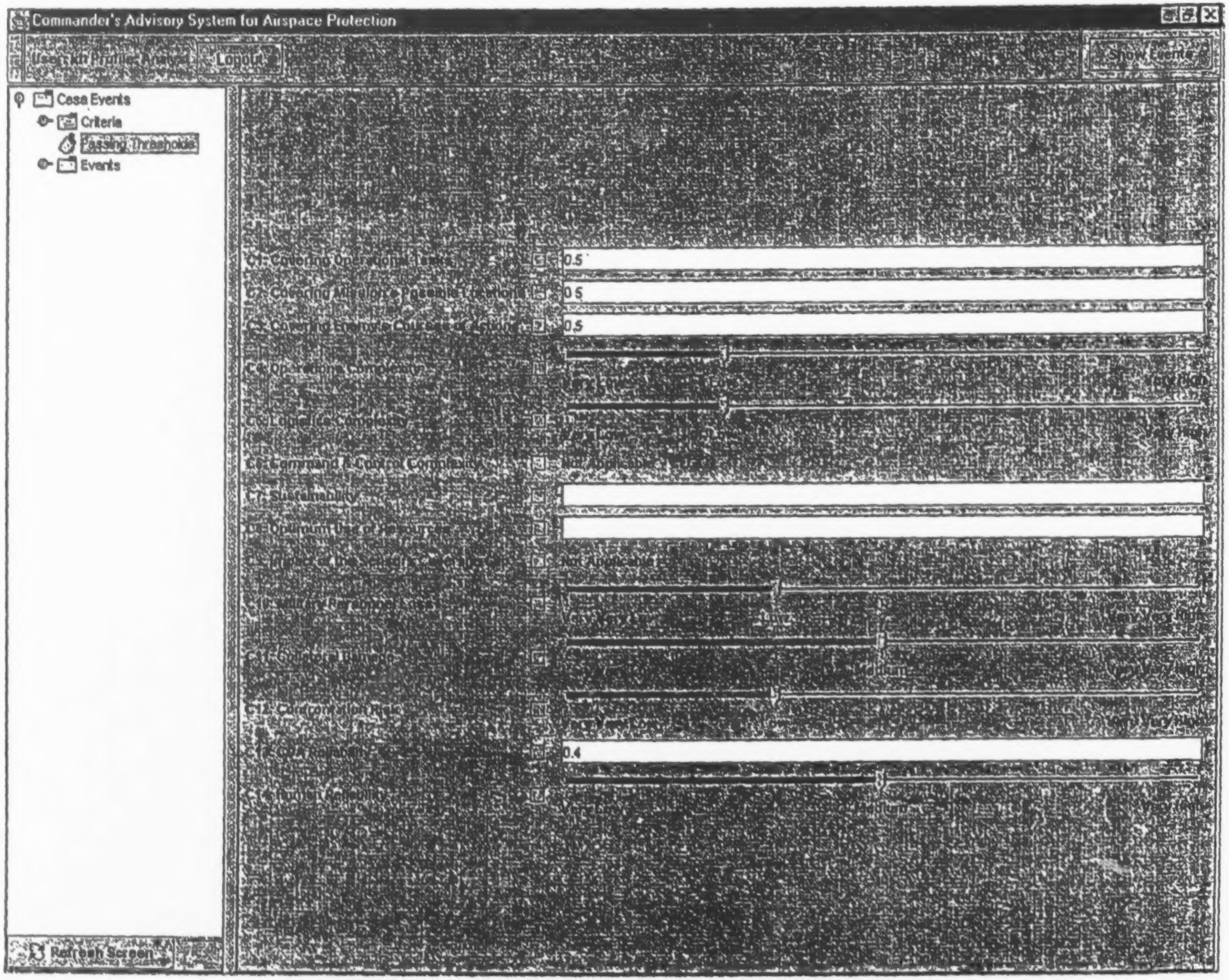
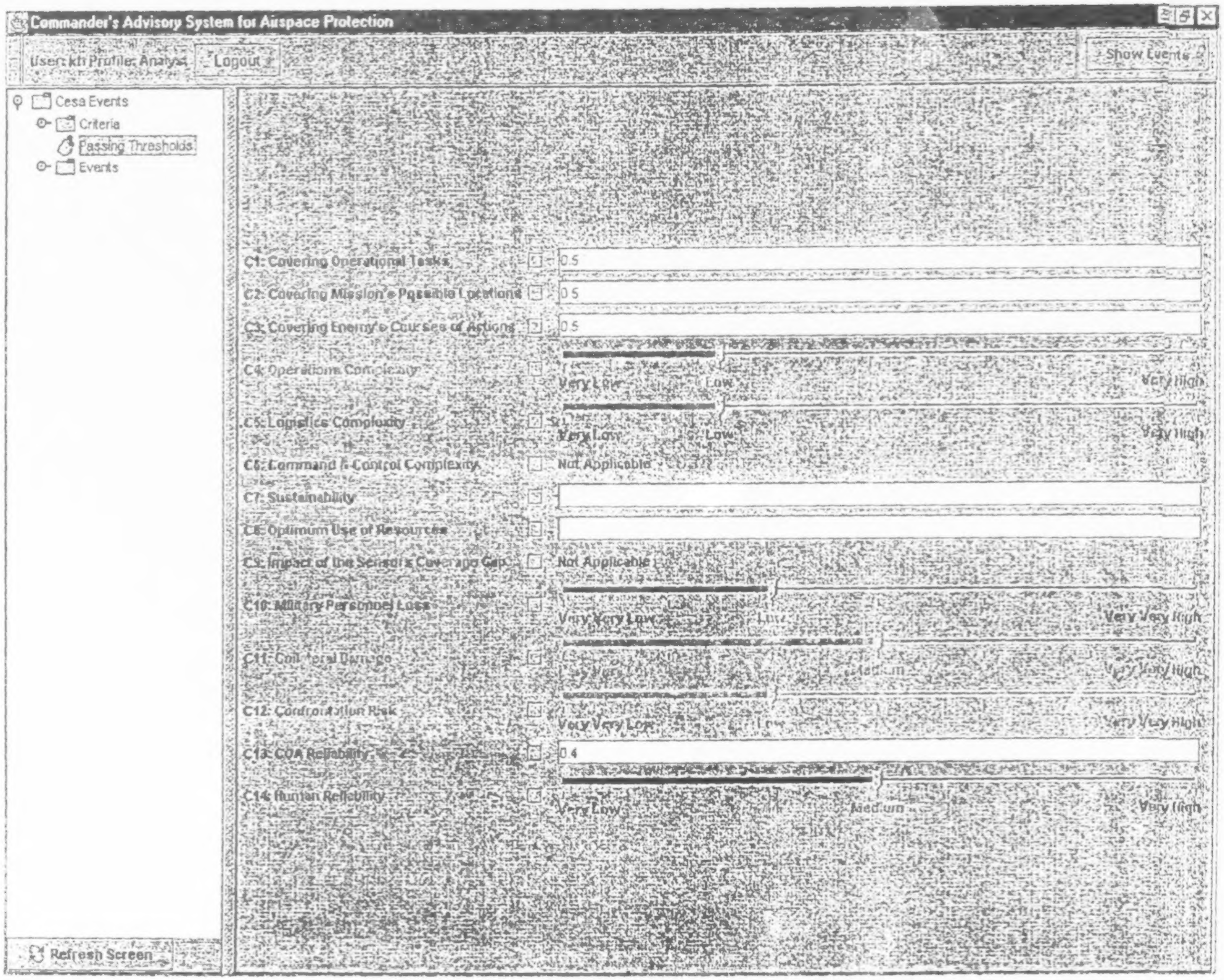

Commander's Advisory System for Airspace Protection
Logout
Show Events
Casa Events
Criteria
Passing Thresholds
Events
Refresh Screen

FIGURE 12 - Example of the CoA screening thresholds


Commander's Advisory System for Airspace Protection
User: kh Profile: Analyst
Logout
Show Events
Cesa Events
Criteria
Passing Thresholds
Events
Refresh Screen

C1: Covering Operational Tasks: 0.5

C2: Covering Mission's Possible Locations: 0.5

C3: Covering Enemy's Courses of Actions: 0.5

C4: Operations Complexity: Very Low – Low – Very High

C5: Logistics Complexity: Very Low – Low – Very High

C6: Command & Control Complexity: Not Applicable

C7: Sustainability:

C8: Optimum Use of Resources:

C9: Impact of the Sensor's Coverage Gap: Not Applicable

C10: Military Personnel Loss: Very Very Low – Low – Very Very High

C11: Collateral Damage: Very Very Low – Medium – Very Very High

C12: Confrontation Risk: Very Very Low – Low – Very Very High

C13: COA Reliability: 0.4

C14: Human Reliability: Very Low – Medium – Very High

FIGURE 12 - Example of the CoA screening thresholds

## 7.0 CONCLUSION

The goal of this technical report was to describe the work done with respect to the selection and the implementation a Multiple Criteria Aggregation Procedure (MCAP) used within the *Commander's Advisory System for Airspace Protection* (CASAP). The aim of this system is to manage counter-drug events and their related courses of actions (CoAs), as well as the evaluation and prioritisation of the CoAs according to the Commander's preferences and concerns.

To be able to determine the most appropriate MCAP for a decision-making situation (DMS) in the context of Canadian airspace protection, we conducted a comparative study to characterise the properties of various discrete MCAPs. This study allowed us to analyse and categorise no less than 30 different reported MCAPs. This study led us to opt for PAMSSEM to be implemented in CASAP. PAMSSEM is a procedure originally proposed by Martel *et al.* [Ref. 31]. It is able to deal with mixed evaluations: of different natures and different types. In fact, the evaluations can be crisp quantities, discrete/continues random, linguistic or fuzzy variables. PAMSSEM also deals with missing evaluations that could be the result of lack of knowledge, forgetfulness or inappropriateness. Furthermore, PAMSSEM uses discriminating thresholds (indifference and strict preference) with different criteria. In order to model the inter-criteria information, PAMSSEM introduces the relative importance coefficient to balance the criteria, as well as veto thresholds for some criteria.

In our opinion, PAMSSEM is one of the very few reported procedures that can handle properly mixed and missing evaluations. This constitutes a major breakthrough within the multiple criteria decision analysis field. In fact, the major reported procedures suppose that the data contained in the multiple criteria performance matrix E is necessarily homogeneous and complete. In other words, these procedures require that the decision-making situation be modelled considering only homogeneous criteria. This is not realistic when dealing with multidimensional and complex decision-making situations. Hence, in our opinion, PAMSSEM brings more flexibility to the modelling and formulation processes (knowledge engineering).

The multiple criteria decision analysis is mostly about considering the decision maker's (Commander) preferences and values within the decision model. The criteria must represent the key issues considered by the Commander's and his senior staff to evaluate different courses of actions to overcome a threat (in this application, we consider the context of counter-drug operations). Moreover, the Commander's preferences are modelled through a set of parameters: discrimination thresholds and criteria relative importance coefficients. Since these parameters are not easy to fix, procedures and guidelines have been developed to ease their setting in CASAP. Actually, methods were proposed to set these thresholds, and to determine default values. A comprehensive graphical criteria-balancing method was developed and implemented to help the DM fix the relative importance coefficient criteria.

To better assist the DM in analysing the alternatives, analysis capabilities have been developed and implemented in CASAP. In particular, a stability analysis and two "*what if*" analysis heuristics were developed. The stability analysis aims at revealing the stability intervals of the criteria relative importance coefficients to preserve the actual ranking. In other words, this type of analysis indicates to what extent the DM can change the criteria "importance" without altering the prioritisation of the CoAs. This is an interesting tool to help discover the more sensitive factors. Effectively, the stability analysis verifies for each selected criterion (factor), to which extent the relative importance coefficient may change without changing the ranking. To our knowledge, this is the first time a global stability analysis is developed for a MCAP of the outranking synthesising approach.

The two "*what if*" analysis heuristics are also developed for CASAP. The first heuristic has been proposed to analyse the possible effects of the evaluations modifications. The motivation of this heuristic is the subjectivity and imperfection (imprecision and uncertainty) of the evaluation processes. From one operator to another, the evaluation of CoA according to an indicator could change. This analysis allows the DM to see the possible effects of any evaluation alteration on the prioritisation of the CoAs (or even on performance of a particular CoA). Here again, in our opinion, this constitutes a powerful tool to make comprehensive and precise decision analysis.

The second "*what if*" analysis heuristic has been proposed to deal with the effects of the thresholds. The thresholds play a major role within the aggregation procedure since they represent the DM's preferences. However, it is an overwhelming task to precisely set these thresholds. By developing a "what if" analysis heuristic for the thresholds, the DM is able to examine the effects of varying his indifference and/or preference thresholds on the outcomes of the prioritisation process. He/she can also consider the effect of the veto thresholds.

This work allowed to better defining the types of decision aid the AOC staff could be provided with. There is still a lot of validation with the operations people to be achieved before being able to qualify or quantify the advantages of such tool. The criteria and their settings have to be validated to be sure that they represent a coherent family of criteria for the DM. The analysis approaches defined in this report also have to be fully validated with users to investigate the understanding of the DM of these tools and improve/adapt them to his requirements. These validations will allow completing the prototype, which will be used to demonstrate the potentials and the effectiveness of such tools in an operational environment. It is always possible to enhance and improve the prototype to better serve the AOC community, and meet the Air Force C&C requirements.

## 8.0 ACKNOWLEDGMENTS

The authors would like to thank Messrs. K. Jabeur for the development of the PAMSSEM calculation module, K. El-hage and C. Gauthier for the development of the analysis tools, as well as Ms. S. Leclerc for the development of some basic user interfaces.

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# APPENDIX A – MCAP DESCRIPTION

## TABLE VI

Single criterion synthesising procedures

| Procedures | Reference | A brief description of the procedure |
|---|---|---|
| Weighted sum | [Ref. 49] | This procedure chooses as the best alternative the one which obtains the best global performance computed using a weighted sum of the performances of the alternatives along each criterion. |
| Fuzzy weighted sum | [Ref. 50] | This procedure is a fuzzified version of the weighted sum in which the weights and the performances are modelled as fuzzy numbers. |
| TOPSIS | [Ref. 49] | This procedure chooses as the best alternative the one which is closest to an ideal solution and furthest from a worst solution. |
| SMART | [Ref. 51] | This is an implementation of MAUT where the global utility function of each alternative is approximated by a weighted linear average of individual evaluations. |
| MAVT | [Ref. 13] | This procedure ranks the alternatives $a_i$ using a global value function V($a_i$) based on partial value functions defined for each criterion and evaluated for each alternative. |
| UTA | [Ref. 52] | This is an implementation of MAVT where individual value functions (for each criterion) are obtained using ordinal regression. |
| MAUT | [Ref. 13] | This is a similar to MAVT where the value functions (partial and global) are utility functions which necessitate stricter hypotheses than MAVT. |
| AHP | [Ref. 16] | A problem is decomposed into a hierarchy where the alternatives are at the lowest level. This technique applies the decomposition, the comparative judgements on comparative elements and measures of relative importance through pairwise comparison matrices which are recombined into an overall rating of alternatives. |
| EVAMIX | [Ref. 54] | This method ranks the alternatives based on a global dominance index, computed for pairs of alternatives, built from an ordinal dominance index and a cardinal one. |

## TABLE VII

## Outranking synthesising procedures

| Procedure | Reference | A brief description of the procedure |
|---|---|---|
| ELECTRE I | [Ref. 15] | The concept of outranking relationship is used. The procedure seeks to reduce the size of non-dominated set of alternatives (kernel). The idea is that an alternative can be eliminated if it is dominated by other alternatives to a specific degree. This is done first by obtaining an outranking relation following pairwise comparisons of alternatives on every criterion, and then by exploiting this outranking relation in order to obtain this sub-set. |
| ELECTRE IS | [Ref. 15] | This is ELECTRE I + discriminating thresholds. |
| ELECTRE II | [Ref. 15] | This is procedure leads to two types of outranking relations: weak $S_f$ and strong $S^F$. |
| ELECTRE III | [Ref. 15] | In this version of ELECTRE, a credibility index, modeled as a fuzzy number, is used to associate a value to the outranking relation. |
| ELECTRE IV | [Ref. 15] | No weights are assigned to the criteria in this version. One index of credibility may be assigned to each of the four outranking relations, $S^1$, $S^2$, $S^3$, $S^4$. |
| MELCHIOR | [Ref. 55] | This is an extension of ELECTRE IV in which there exists an order relation reflecting the relative importance of the criteria. |
| ELECTRE TRI | [Ref. 15] | This is customized for a sorting decision problem. It uses conjunctive and disjunctive techniques to assign different alternatives to different categories. |
| PROMÉTHÉE I | [Ref. 33] | PROMETHEE I is based on the same principles as ELECTRE and introduces six functions to describe the DM preferences along each criterion. This procedure provides a partial order of the alternatives using entering and leaving flows. For an alternative $a_i$, an entering flow represents the importance of the alternatives outranking $a_i$, and a leaving flow represents the importance of the alternatives outranked by $a_i$. |
| PROMÉTHÉE II | [Ref. 55] | PROMETHEE II is based on the same principles as PROMETHEE I. This procedure provides a total preorder of the alternatives using an aggregation of the entering and leaving flows to obtain a "global score" for each alternative. |
| ORESTE | [Ref. 56] | This procedure provides an outranking relation |

| Procedure | Reference | A brief description of the procedure |
|---|---|---|
| | | by using as input ordinal evaluations of the alternatives and the ranking of the criteria as a function of their relative importance. |
| REGIME | [Ref. 57] | A pairwise comparison of the alternatives assigns the value 1 to the dominating alternative and -1 to the dominated one, or 0 if there is indifference between the two. This matrix is then exploited to obtain a recommendation. |
| NAIADE | [Ref. 34] | This procedure uses a distance operator to obtain the pairwise evaluations which are modeled as fuzzy numbers. The aggregation produces as in PROMÉTHÉE incoming and outgoing flows. |
| PAMSSEM I and II | [Ref. 31] | See Chapter 3 of this report. |

## TABLE VIII

## Mixed procedures

| Procedures | Reference | A brief description of the procedure |
|---|---|---|
| Maximin Procedure | [Ref. 49] | This procedure selects the alternative which ranks highest among the lowest ranking ones. This is a conservative approach (the strongest of the weakest). |
| Fuzzy Maximin Procedure | [Ref. 58] | This procedure is an adaptation of the maximum where the scores are modelled by fuzzy numbers. |
| Lexicographic Method | [Ref. 49] | This procedure first ranks the criteria in decreasing importance and chooses the alternative which performs best on the most important criterion. |
| Disjunctive Method | [Ref. 49] | This procedure sets high thresholds for each criterion and retains all the alternatives which score above these thresholds on one or more criteria. |
| Conjunctive Method | [Ref. 49] | This procedure uses minimal acceptable levels for each criterion and rejects all the alternatives which do not meet the minimal requirements for all the criteria. |
| Fuzzy Conjunctive/ Disjunctive | [Ref. 59] | This is an adaptation of the crisp conjunctive/disjunctive methods where the evaluations are fuzzy numbers. |
| QUALIFEX | [Ref. 60] | This procedure uses successive permutations of the order of alternatives in order to obtain a ranking consistent with the ordinal evaluations provided. |
| Zaras and Martel Method | [Ref. 61] | This method is appropriate when the evaluations are probability distributions and uses the notion of stochastic dominance. |

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A multicriteria aggregation procedure for the evaluation of courses of action in the context of the Canadian airspace protection.

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| | | |
|---|---|---|
| 5. DATE OF PUBLICATION (month and year)<br>February 2001 | 6a. NO. OF PAGES<br>70 | 6b .NO. OF REFERENCES<br>61 |

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**ABSTRACT**

Defence Research Establishment Valcartier initiated a research activity aimed at investigating and developing advanced technologies, approaches and concepts to provide the 1CAD/CANR Air Operation Centre (AOC) Commander and his senior staff, with advisory tools for planning, management and employment of air defence resources and capabilities. An advisor tool was designed to assist the AOC staff managing events and their related Courses of Action (CoAs), as well as prioritising these CoAs according to different evaluation criteria. This command and control tool, called Commander's Advisory System for Airspace Protection (CASAP), was foremost developed to deal with events of counter-drug operations: to intercept drug smugglers violating the Canadian airspace, in co-ordination with different services and parties. Since the Commander needs to balance several conflicting and incommensurable criteria to make "wise" decisions in such situation, Multicriteria Decision Aid models and procedures were deemed to be appropriate to deal with Canadian aerospace protection decision-making situations. This document, one of three, presents the multicriteria aggregation procedure and the associated analysis tools that were implemented for this advisor tool. The two other documents will present the decision-making situations structuring process (including the formulation of the different criteria), as well as the functional architecture of CASAP.

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